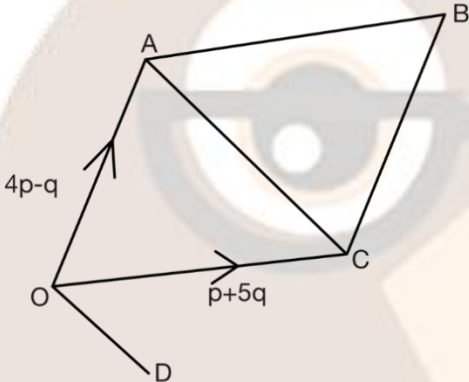


6 ESSENTIAL QUESTIONS TO MASTER

VECTORS

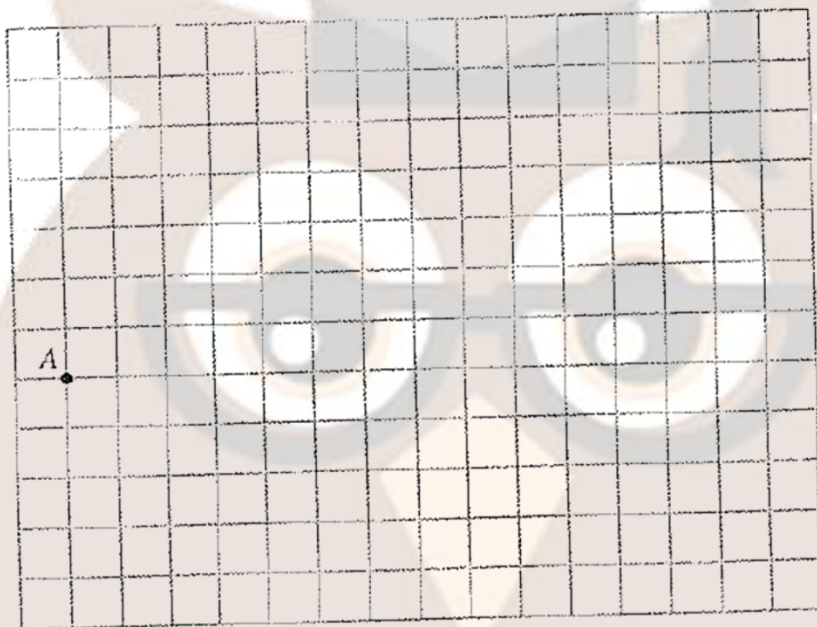
1	In the diagram, OABC is a parallelogram, $\overrightarrow{OA} = 4p - q$ $\overrightarrow{OC} = p + 5q$  (a) Express, as simply as possible, in terms of p and q: (i) $\overrightarrow{BC}$ (ii) $\overrightarrow{AC}$ (b) D is the point such that $\overrightarrow{OD} = -p + 2q$. (i) Explain why $\overrightarrow{AC}$ is parallel to $\overrightarrow{OD}$. (ii) Given that the area of triangle OAC is 18 square units, find the area of triangle OCD.	
2	O is the origin, L is the point (5, -2) and M is the point (2, 3). Given that the vector $\overrightarrow{OA}$ is such that $\overrightarrow{OA} = \overrightarrow{OL} + 2\overrightarrow{OM}$, find (a) the coordinates of A (b) the value of $\overrightarrow{OM}$, correct to two significant figures.	
3	$\overrightarrow{AB} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ (a) Express $\overrightarrow{AC}$ as a column vector.	

(b)

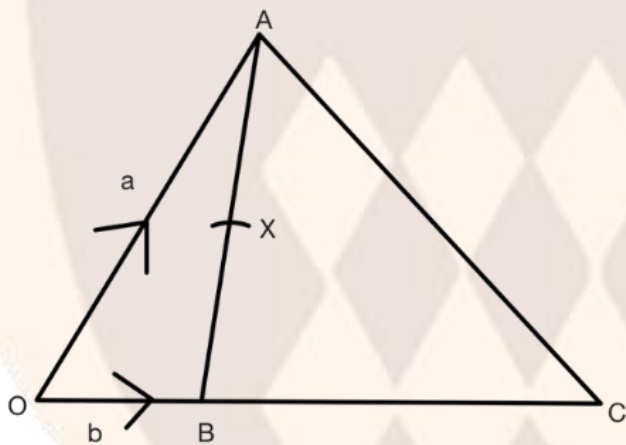
$$\overrightarrow{CD} = \begin{pmatrix} -11 \\ h \end{pmatrix}$$

Find the two possible values of h which will make ABCD a trapezium.

You may use the grid below to help you with your investigation.



4

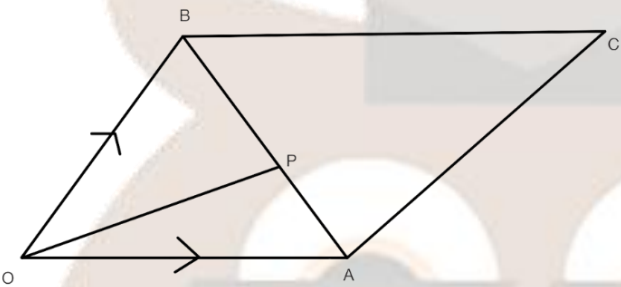


OAC is a triangle and B is a point on OC.

$\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$ and $OB:BC = 2:3$

X is the point on AB such that $AX : XB = 1 : 2$

(a) Express $\overrightarrow{AC}$ in terms of a and b , as simply as possible.

	(b) Express $\overrightarrow{XB}$ in terms of a and b, as simply as possible. (c) Y is the point on OC such that AXYC is a trapezium. Find, in terms of a and b, $\overrightarrow{XY}$.	
5	In the diagram, $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$, $\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{BC} = 3\overrightarrow{OA}$  (i) Express each of the following in terms of a and b. (a) $\overrightarrow{OC}$ (b) $\overrightarrow{AP}$ (c) $\overrightarrow{OP}$ (ii) What can you deduce about the points O, P and C? (iii) Find $\frac{\text{area of triangle } OAP}{\text{area of triangle } OBP}$	
6	PQRS is a parallelogram. P is the point (-8,-2) Q is the point (-6,2) and $\overrightarrow{QR} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ (i) Find the vector $\overrightarrow{PQ}$ (ii) Use vectors to show whether the point A(-11,7) lies on the line PQ. (iii) Find the length of PR. (iv) Find the equation of the line RS.	

Answer Key

1	(a) (i) $\overrightarrow{BC} = -4p + q$. (ii) $\overrightarrow{AC} = -3p + 6q$	
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	(b) (i) $\overrightarrow{OD} = -p+2q$, $\overrightarrow{AC} = 3(-p + 2q)$ $\overrightarrow{AC}=3\overrightarrow{OD}$ Hence $\overrightarrow{AC}$ is a scalar multiple of $\overrightarrow{OD}$, so they are parallel. (ii) The area of triangle OCD is 6 square units.	
2	(a) 1. Write down the vectors: $\overrightarrow{OL}=(5,-2)$ $\overrightarrow{OM}=(2,3)$ 2. Compute $\overrightarrow{OM}$: $2(2,3) = (4,6).$ 3. Subtract from $\overrightarrow{OL}$:= $\overrightarrow{OA} = (5, -2) - (4,6)$ $= (5 - 4, -2 - 6)$ $= (1, -8).$ Thus A has coordinates $(1, -8)$. (b) 1. We have $\overrightarrow{OA} = (1,-8)$ 2. Its length is $ \begin{aligned} \overrightarrow{OA} &= \sqrt{1^2 + (-8)^2} \\ &= \sqrt{1 + 64} \\ &= \sqrt{65} \\ &\approx 8.0623 \dots \end{aligned} $ 3. Rounding to two significant figures gives 8.1.	
3	(a) $\begin{pmatrix} 14 \\ 0 \end{pmatrix}$ (b) Call $\overrightarrow{DA}$ the fourth side. Since walking all the way around the quadrilateral must bring you back to the start, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0$ $\overrightarrow{DA} = -(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}) = -\begin{pmatrix} 8 + 6 - 11 \\ -4 + 4 + h \end{pmatrix} = -\begin{pmatrix} 3 \\ h \end{pmatrix} = \begin{pmatrix} -3 \\ -h \end{pmatrix}$	

Now there are two ways to get exactly one pair of parallel sides:

1. Make $(BC \parallel DA)$.

Two vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ are parallel exactly when one is a scalar multiple of the other, say $\begin{pmatrix} c \\ d \end{pmatrix} = k \begin{pmatrix} a \\ b \end{pmatrix}$

Here we set $\overrightarrow{DA} = k\overrightarrow{BC}$. So

$$\begin{pmatrix} -3 \\ -h \end{pmatrix} = k \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

Equating components gives

$$-3 = 6k \Rightarrow k = -\frac{1}{2}$$

and

$$-h = 4k = 4\left(-\frac{1}{2}\right) = -2$$

$$\Rightarrow h = 2.$$

2. Make $(AB \parallel CD)$. Set

$$\overrightarrow{CD} = m\overrightarrow{AB}$$

$$\begin{pmatrix} -11 \\ h \end{pmatrix} = m \begin{pmatrix} 8 \\ -4 \end{pmatrix}$$

Then

$$-11 = 8m \Rightarrow m = -\frac{11}{8},$$

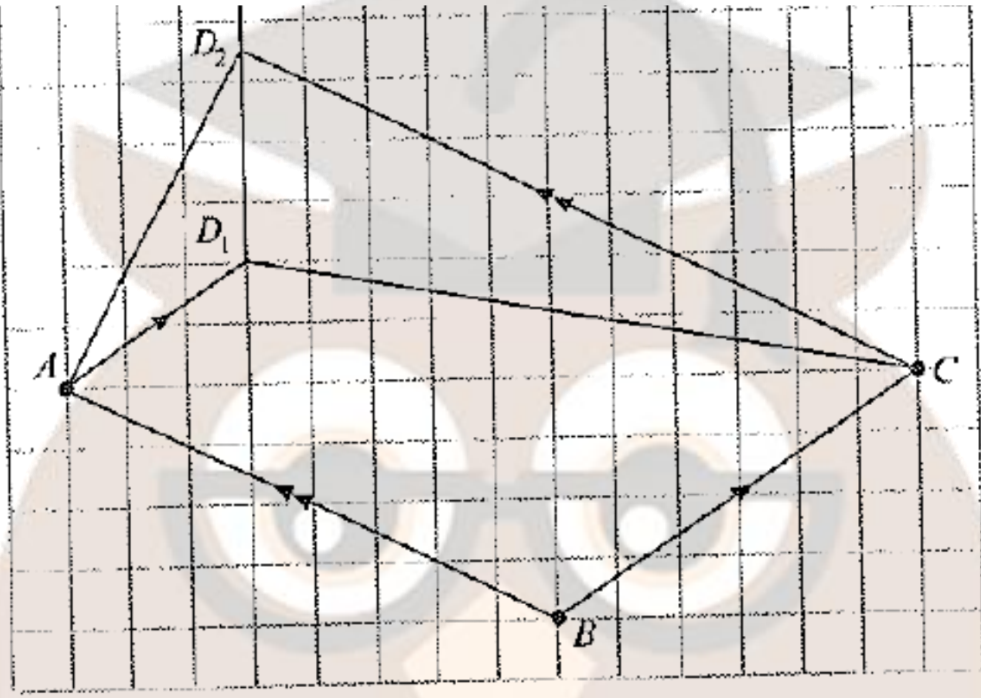
and

$$h = -4m = -4\left(-\frac{11}{8}\right)$$

$$= \frac{44}{8}$$

$$= 5.5$$

These are the only two ways to get exactly one pair of parallel sides, so the two possible values of h are 2 and 5.5.

		
4	(a) $\overrightarrow{OC} = 2.5b$ $\overrightarrow{AC} = 2.5b - a$ (b) $\overrightarrow{AB} = b - a$ $\overrightarrow{XB} = \frac{2}{3}(b - a)$ (c) For AXYC to be a trapezium, triangle BXY is similar to triangle BAC. $\frac{XY}{AC} = \frac{BX}{BA} = \frac{2}{3}$ $\overrightarrow{XY} = \frac{2}{3}\overrightarrow{AC}$ $\frac{2}{3}(\frac{5}{2}b - a)$ $\frac{5}{3}b - \frac{2}{3}a$	
5	(i) (a) $\overrightarrow{OC} = b + 3a$ (b) $\overrightarrow{AP} = \frac{1}{4}(b - a)$ (c) $\overrightarrow{OP} = \frac{1}{4}(b + 3a)$ (ii) Since $\overrightarrow{OP} = \frac{1}{4}(b + 3a) = \frac{1}{4}\overrightarrow{OC}$, point P lies on the line OC. Hence O, P, C are collinear.	

	(iii) $\frac{\text{area of } \triangle OAP}{\text{area of } \triangle OBP}$ $= \frac{1}{3}$	
6	(i) $PQ = \begin{pmatrix} -6 \\ 2 \end{pmatrix} - \begin{pmatrix} -8 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (ii) $PQ = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $PA = \begin{pmatrix} -11 \\ 7 \end{pmatrix} - \begin{pmatrix} -8 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 9 \end{pmatrix}$ $= 3 \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ PQ is not parallel to PA. A does not lie on the line PQ. (iii) $PR = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ means $PQ + QR$ $= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ $PR = \sqrt{6^2 + 4^2}$ $= 7.21 \text{ units}$ (iv) $OR = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ $R = (-2, 2)$ $m_{PQ} + m_{RS}$ $= 4 \div 2$ $= 2$ Equation $y = mx + c$: $2 = 2(-2) + c$ $\Rightarrow c = 6$ Hence the line is $y = 2x + 6$	