

7 ESSENTIAL QUESTIONS TO MASTER

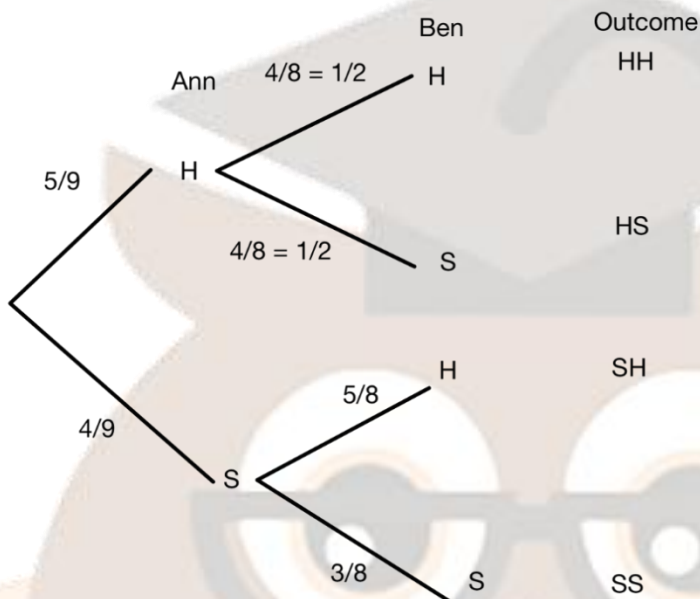
PROBABILITY

1	A box of chocolates contains 5 chocolates with hard centres and 4 with soft centres. Ann takes a chocolate, selected at random, from the box and eats it. Ben then takes a chocolate, selected at random, from the box. (a) Draw a tree diagram to show the probabilities of the possible outcomes. (b) Find, as a fraction in its simplest form, the probability that (i) Ann and Ben both choose a chocolate with a hard centre, (ii) Ben chooses a chocolate with a soft centre, (iii) one of them chooses a chocolate with a hard centre and the other chooses one with a soft centre.	
2	A bag contains five counters, numbered 1, 2, 3, 4 and 5. Two counters are taken from the bag at random, one after the other, without replacement. (a) Draw a possibility diagram to represent the outcomes. (b) Find, in its simplest form, the probability that (i) both counters have numbers less than 3, (ii) neither counter has an even number, (iii) the sum of the numbers is 10, (iv) the product of the numbers is less than 6.	
3	There are $(2n + 3)$ red marbles, $(4n - 1)$ green marbles and $(18 - 2n)$ blue marbles in a bag. (i) Given that the probability of drawing a red or green marble is $\frac{19}{22}$, show that $n = 6$. (ii) Hence find the probability of drawing 2 red marbles, one after another without replacement.	
4	A bag contains 5 red, 7 blue and 3 yellow identical balls. (i) One ball is selected at random. Find the probability that it is a yellow ball. (ii) Two balls are selected at random without replacement. Find the probability that both balls are of different colours. (iii) Three balls are selected at random, with replacement. Find the probability that two red balls and one blue ball are chosen.	
5	During a game, Lee rolled two fair six-sided dice. To obtain the score, he subtracted the lower number from the higher number. If the numbers shown on the dice were the same, his score is zero.	

	(a) Construct a possibility diagram to show all possible outcomes. (b) Find the probability that Lee's score is 5.										
6	The table below shows the languages studied by 30 students. There are seven students who only study Spanish but not French. One of the 30 students is selected at random. The probability that the student studies French and not Spanish is $\frac{4}{15}$ Two of the 30 students are selected at random. The probability that both students study neither French nor Spanish is $\frac{1}{145}$ Complete the table of information about the 30 students. <table border="1"> <thead> <tr> <th></th><th>French</th><th>Not French</th></tr> </thead> <tbody> <tr> <th>Spanish</th><td></td><td></td></tr> <tr> <th>French</th><td></td><td></td></tr> </tbody> </table>		French	Not French	Spanish			French			
	French	Not French									
Spanish											
French											
7	25 cards bearing the numbers 1,2,3,...,24,25 are put inside a box A. (a) A card is picked from box A at random. Find the probability that the selected card is (i) a multiple of 3 as well as a multiple of 7 (ii) a multiple of 3 or a multiple of 7. (b) Cards which are either a multiple of 3 or a multiple of 4 are selected and put in box B. A card is then picked at random from box B. Find the probability that the selected card is (i) a multiple of 7 (ii) a multiple of 3, 4 or 7.										

Answer Key

1	(a) Let H = hard centre and S = soft centre.	
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(b) (i) Probability Ann and Ben both choose a hard-centre chocolate = $P(HH)$

$$= \frac{5}{9} \times \frac{1}{2}$$

$$= \frac{5}{18}$$

(ii) Probability that Ben chooses a soft-centre chocolate = $P(HS \text{ or } SS)$

$$= \left(\frac{5}{9} \times \frac{1}{2} \right) + \left(\frac{4}{9} \times \frac{3}{8} \right)$$

$$= \frac{4}{9}$$

(iii) Probability that one of them picks hard and the other picks soft = $P(HS \text{ or } SH)$

$$= \left(\frac{5}{9} \times \frac{1}{2} \right) + \left(\frac{4}{9} \times \frac{5}{8} \right)$$

$$= \frac{5}{9}$$

2

(a)

1st Counter

	1	2	3	4	5
1		(2,1)	(3,1)	(4,1)	(5,1)
2	(1,2)		(3,2)	(4,2)	(5,2)
3	(1,3)	(2,3)		(4,3)	(5,3)
4	(1,4)	(2,4)	(3,4)		(5,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	

2nd
Counter

(b)(i) $P(\text{both counters have numbers less than 3}) = \frac{2}{20} = \frac{1}{10}$

	(ii) $P(\text{Neither counter has an even number}) = \frac{6}{20} = \frac{3}{10}$ (iii) $P(\text{sum of the numbers is } 10) = 0$ (iv) $P(\text{product of the numbers is less than } 6) = \frac{8}{20} = \frac{2}{5}$								
3	(i) $\frac{2n+3+4n-1}{2n+3+4n-1+18-2n} = \frac{19}{22}$ $\frac{6n+2}{20+4n} = \frac{19}{22}$ $132n+44=380+76n$ $56n=366$ $n=6$ (ii) $\frac{15}{44} \times \frac{14}{43} = \frac{105}{946}$								
4	(i) $\frac{3}{15} = \frac{1}{5}$ (ii) $= 1 - P(\text{both same colours})$ $= 1 - [P(\text{red, red}) + P(\text{blue, blue}) + P(\text{yellow, yellow})]$ $= 1 - \left[\left(\frac{5}{15} \times \frac{4}{14} \right) + \left(\frac{7}{15} \times \frac{6}{14} \right) + \left(\frac{3}{15} \times \frac{2}{14} \right) \right]$ $= 1 - \frac{34}{105}$ $= \frac{71}{105}$ Alternative Method: $P(\text{red, blue}) + P(\text{red, yellow}) + P(\text{blue, red}) + P(\text{blue, yellow}) + P(\text{yellow, red}) + P(\text{yellow, blue})$ $= \left[\left(\frac{5}{15} \times \frac{7}{14} \right) + \left(\frac{5}{15} \times \frac{3}{14} \right) + \left(\frac{7}{15} \times \frac{5}{14} \right) + \left(\frac{7}{15} \times \frac{3}{14} \right) + \left(\frac{3}{15} \times \frac{5}{14} \right) + \left(\frac{3}{15} \times \frac{7}{14} \right) \right]$ $= \frac{71}{105}$								
5	(a) <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>		1	2	3	4	5	6	
	1	2	3	4	5	6			

	<table><tr><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td></tr><tr><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td></tr></table>	1	0	1	2	3	4	5	2	1	0	1	2	3	4	3	2	1	0	1	2	3	4	3	2	1	0	1	2	5	4	3	2	1	0	1	6	5	4	3	2	1	0	
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	(b) $\frac{2}{36}=\frac{1}{18}$																																											
6	Number of students who speak French and not Spanish $\frac{4}{15} \times 30 = 8.$ Let the number of students who speak neither French nor Spanish be (n). The probability that both selected students speak neither French nor Spanish is $\frac{\binom{n}{2}}{\binom{30}{2}} = \frac{1}{145}.$ $\frac{\binom{n}{2}}{\binom{30}{2}} = \frac{1}{145}$ $145(n^2 - n) = 870$ $\Rightarrow$ $145n^2 - 145n - 870 = 0.$ $n^2 - n - 6 = 0$ $\Rightarrow$ $(n - 3)(n + 2) = 0.$ $n=-2$ (NA) or $n=3$ <table><tr><td></td><td>French</td><td>Not French</td></tr><tr><td>Spanish</td><td>12</td><td>7</td></tr><tr><td>French</td><td>8</td><td>3</td></tr></table>		French	Not French	Spanish	12	7	French	8	3																																		
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7	(a) From box A, 25 equally likely cards 1. Total number of cards = 25. (a)(i) a multiple of 3 as well as a multiple of 7 A number is a multiple of both 3 and 7 exactly when it is a multiple of $\text{lcm}(3,7)=21$.																																											

– Multiples of 21 from 1 to 25: 21 only → 1 card. So $P(\text{multiple of 3 and 7}) = \frac{1}{25}.$ (a)(ii) a multiple of 3 or a multiple of 7 – Multiples of 3 in $\{1 \cdots 25\}$: 3, 6, 9, 12, 15, 18, 21, 24 → 8 cards – Multiples of 7 in $\{1 \cdots 25\}$: 7, 14, 21 → 3 cards – Intersection (multiples of both): $\{21\}$ → counted twice if we just add. By inclusion–exclusion, $\{3 \text{ or } 7\} = 8 + 3 - 1 = 10.$ Hence $P(\text{multiple of 3 or 7}) = \frac{10}{25} = \frac{2}{5}.$ (b) Forming box B and drawing from it First form B = {cards that are multiples of 3 or multiples of 4}. Multiples of 3 in $\{1 \cdots 25\}$: 3, 6, 9, 12, 15, 18, 21, 24 → 8 cards Multiples of 4 in $\{1 \cdots 25\}$: 4, 8, 12, 16, 20, 24 → 6 cards Intersection (multiples of 12): 12, 24 → 2 cards So by inclusion–exclusion, $ B = 8 + 6 - 2 = 12.$ We now pick one card uniformly from these 12. (b)(i) a multiple of 7 in box B None of the multiples of 7 (7, 14) are in B, except 21 which is also a multiple of 3 and so is in B. So exactly one card (21) is a multiple of 7. $P(\text{multiple of 7} \mid B)$	
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$= \frac{1}{12}$ (b)(ii) a multiple of 3, 4 or 7 in box B But B itself consists only of multiples of 3 or 4, and 21 (the lone multiple of 7) is already in B as a multiple of 3. Thus, every card in B is a multiple of 3 or 4, hence also of 3 or 4 or 7. $P(\text{multiple of 3,4 or 7} \mid B)$ $= \frac{12}{12} = 1.$	
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