

5 ESSENTIAL QUESTIONS TO MASTER

MATRICES

1	Given that $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & p \\ q & 2 \end{pmatrix}$ find (a) A^2 (b) p and q if $AB = 5B$																	
2	A school studio sells three types of tickets—Bronze, Silver and Gold for its weekend musical. The table below shows the number of tickets sold on Saturday and Sunday, and the price of each ticket type. <table><tr><td></td><td>Bronze</td><td>Silver</td><td>Gold</td></tr><tr><td>Saturday</td><td>80</td><td>50</td><td>30</td></tr><tr><td>Sunday</td><td>70</td><td>45</td><td>25</td></tr><tr><td>Price</td><td>\$15</td><td>\$20</td><td>\$25</td></tr></table> (a) Write down two matrices whose product gives the total sales (in dollars) for Saturday and for Sunday. (b) Compute this matrix product to find the sales figures for each day. (c) Write down three matrices whose product gives the total sales for the entire weekend in one shot.		Bronze	Silver	Gold	Saturday	80	50	30	Sunday	70	45	25	Price	\$15	\$20	\$25	
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3	Henry is a yoga instructor. He offers sessions for Basic and Intermediate students on weekdays and at weekends. Each student has a 10-week block of sessions with one session per week. The matrix S shows the number of students he coaches each week in one 10-week block. Basic Intermediate $S = \begin{pmatrix} 6 & 9 \\ 3 & 5 \end{pmatrix} \begin{cases} \text{Weekday} \\ \text{Weekend} \end{cases}$ (a) Evaluate the matrix $T = 10S$. (b) Henry charges \$50 for each Basic session and \$80 for each Intermediate session. Represent the session charges in a (2×1) column matrix F. (c) Evaluate the matrix $Q = TF$. (d) State what the elements of Q represent. (e) Henry wants to attract more students, so in the next 10-week block he reduces his prices by 10%. For this block of sessions, on weekdays he has 10 Basic students and 5 Intermediate students. On weekends he has 6 Basic students and 4 Intermediate students.																	

Rayyen bought 4 litres of water, 2 litres of milk and 3 litres of juice. Zinnie bought 3 litres of water and 4 litres of juice. Represent their purchases in a 2×3 matrix P. (b) Evaluate the matrix $R = PQ$. (c) State what the elements in the second column of matrix R represent. (d) Rayyen shopped in grocery mart H. He got a discount coupon that entitled him to a discount of 15%. How much did he pay altogether for his items?	
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Answer Key:

1	Compute (A^2): $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ $\Rightarrow A^2 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ $= \begin{pmatrix} 7 & 18 \\ 6 & 19 \end{pmatrix}.$ 2. Find p,q so that $AB=5B$: First, $B = \begin{pmatrix} 1 & p \\ q & 2 \end{pmatrix}$ AB $= \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & p \\ q & 2 \end{pmatrix}$ $= \begin{pmatrix} 2+3q & 2p+6 \\ 1+4q & p+8 \end{pmatrix}.$ We require $AB=5B$ $= 5 \begin{pmatrix} 1 & p \\ q & 2 \end{pmatrix}$ $= \begin{pmatrix} 5 & 5p \\ 5q & 10 \end{pmatrix}.$ Equate entries:
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	$-(2 + 3q = 5) (3q = 3) (q = 1).$ $-(2p + 6 = 5p) (6 = 3p) (p = 2).$ Check the other two entries: - $(1 + 4q = 1 + 4 \times 1 = 5 = 5q)$ - $(p + 8 = 2 + 8 = 10)$ Thus $A^2 = \begin{pmatrix} 7 & 18 \\ 6 & 19 \end{pmatrix},$ $(p, q) = (2, 1).$
2	(a) Choosing the two matrices $A = \begin{pmatrix} 80 & 50 & 30 \\ 50 & 45 & 25 \\ 15 & 20 & 25 \end{pmatrix}$ Then the product AP is a (2×1) matrix whose entries are the total sales for Saturday (first row) and Sunday (second row). (b) Evaluating AP $A = \begin{pmatrix} 80 & 50 & 30 \\ 50 & 45 & 25 \\ 15 & 20 & 25 \end{pmatrix}$ $P = \begin{pmatrix} 20 \\ 25 \end{pmatrix}$ When combining together, it becomes $\begin{pmatrix} 2950 \\ 2575 \end{pmatrix}$ (c) Three-matrix product for weekend total We want a single number giving the sum of Saturday + Sunday sales. One way is to prepend the row vector $((1 \ 1))$ to pick and add the two entries of AP. So define $R = (1 \ 1)$ $A = \begin{pmatrix} 80 & 50 & 30 \\ 50 & 45 & 25 \\ 15 & 20 & 25 \end{pmatrix} \quad P = \begin{pmatrix} 20 \\ 25 \end{pmatrix}$ Then $R(AP)$ $= (1 \ 1)$ $= 2950 + 2575$ $= 5525.$

3	(a) $(T = 10S = 10 \begin{pmatrix} 6 & 9 \\ 3 & 5 \end{pmatrix})$ $= \begin{pmatrix} 60 & 90 \\ 30 & 50 \end{pmatrix}$ (b) Let F be the (2×1) column matrix of session charges: $F = \begin{pmatrix} 50 \\ 80 \end{pmatrix}$ (c) $Q = \begin{pmatrix} 60 & 90 \\ 30 & 50 \end{pmatrix} \begin{pmatrix} 50 \\ 80 \end{pmatrix}$ $\begin{pmatrix} 3000 + 7200 \\ 1500 + 4000 \\ 10200 \\ 5500 \end{pmatrix}$ (d) Henry collected for 10-week block sessions on weekdays and at weekends respectively. (e) New prices after a 10% reduction: - Basic: $(50 \times 0.9 = 45)$ dollars per session - Intermediate: $(80 \times 0.9 = 72)$ dollars per session Student numbers per week: - Weekdays: 10 Basic, 5 Intermediate - Weekends: 6 Basic, 4 Intermediate Weekly revenue: - Weekdays: $(10 \times 45 + 5 \times 72 = 450 + 360 = 810)$ - Weekends: $(6 \times 45 + 4 \times 72 = 270 + 288 = 558)$ Total for 10 weeks: $((810 + 558) \times 10 = 1368 \times 10 = 13680.)$
4	$\begin{pmatrix} 2 & 6 \\ -2 & -3 \\ 5 & 1 \end{pmatrix}$ (a)(i) (ii) The change (or students could use “increase or decrease”) in new subscriptions between May

	and June which Aldrick and Bryan obtained for packages F, G and H respectively. (b)(i)30 45 60 (ii)2640 2955 (iii)The total sales commissions Aldrick and Bryan received respectively in May.																																				
5a)	<table><tr><td></td><td>W</td><td>M</td><td>J</td><td></td></tr><tr><td>P=</td><td>4</td><td>2</td><td>3</td><td>Rayyen</td></tr><tr><td></td><td>3</td><td>0</td><td>4</td><td>Zinnie</td></tr></table>		W	M	J		P=	4	2	3	Rayyen		3	0	4	Zinnie																					
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5c)	They represent the difference in the total spendings of Rayyen and of Zinnie at grocery mart G and H respectively for water, fresh milk and juice.																																				
5d)	Amount Rayyen spent $19.90 - 0.30 = \$19.60$ Amount to pay after 15% discount $19.60 \times 85\% = \$16.66$																																				