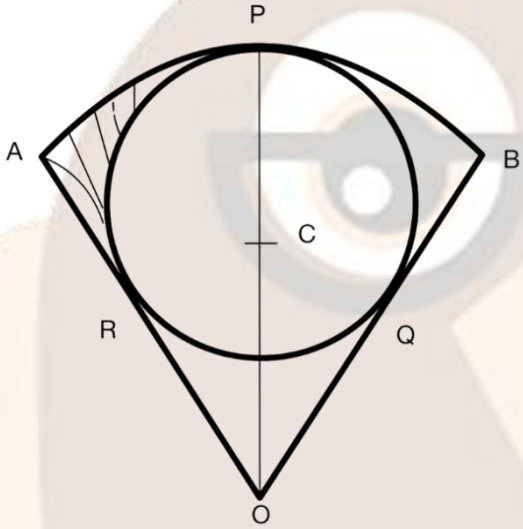
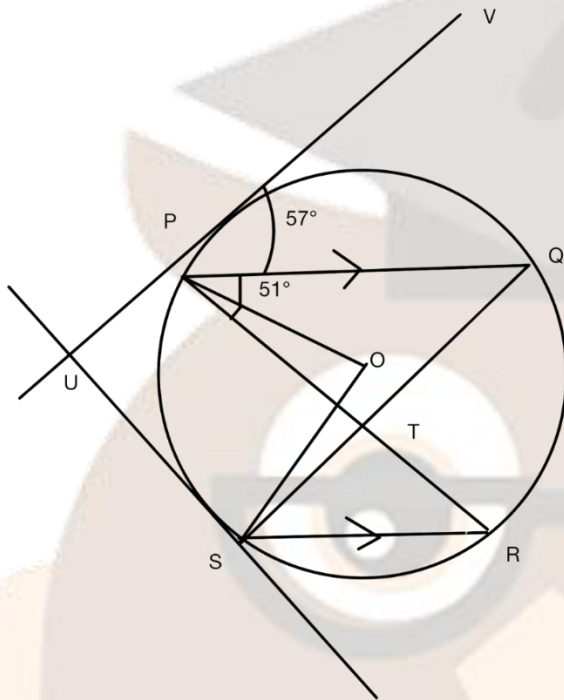


6 ESSENTIAL QUESTIONS TO MASTER

CIRCLES

1	In the diagram, OAPB is a sector of a circle with centre O and radius 9 cm. Angle AOB = 1.4 radians. C is the centre of the circle enclosed inside the sector. OCP is a straight line and the circle touches the sector at points P, Q and R.  (i) Show that the radius of the enclosed circle is 3.526 cm, correct to 3 decimal places. (ii) Calculate the area of the shaded region.	
2	A circle PQRS with centre O has parallel chords PQ and SR. The chord PR intersects the chord SQ at T. The tangents produced from points P and S meet at U. Angle(QPR = 51°) and angle (QPV = 57°).	



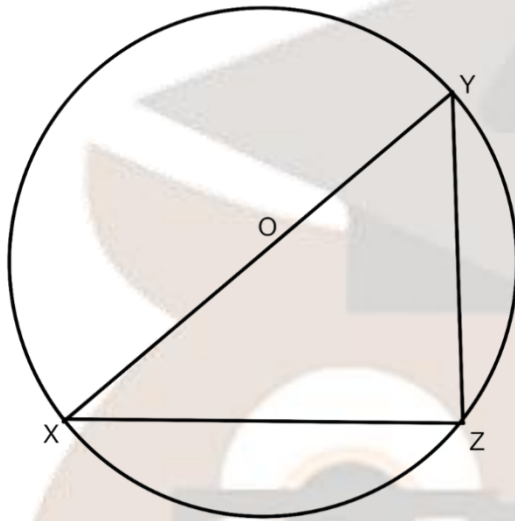
- (i) Find angle POS.
- (ii) Show that triangle PQT is an isosceles triangle.
Give a reason for each statement you make.
- (iii) Explain if a circle can be formed by passing through the points O, P, U and S. State your reason clearly.

- | | |
|---|---|
| 3 | The lines ABC and CDE are tangents to the circle BPDQR at B and at D respectively. The centre of the circle is O. The straight lines POR and BTD intersect at T. It is given that $\angle ACD = 42^\circ$ and $\angle PBC = 31^\circ$. |
|---|---|

Giving all the relevant properties, find

- (a) $\angle DBP$
(b) $\angle RBD$
(c) $\angle DQR$

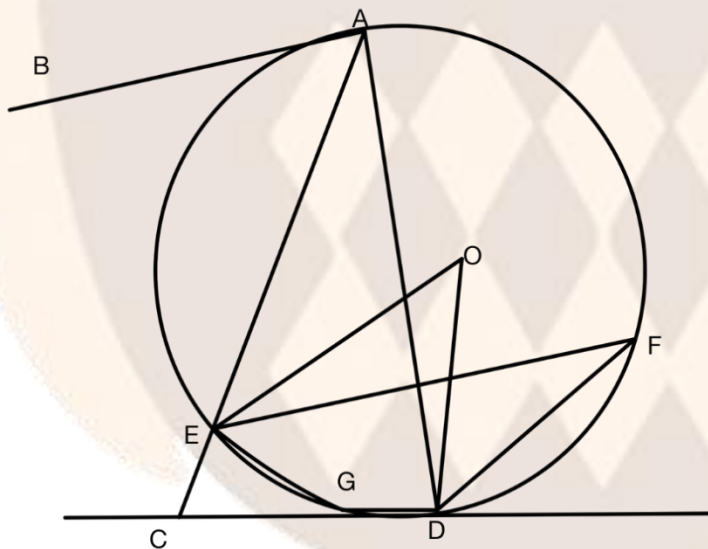
- | | |
|---|--|
| 4 | <p>In the diagram below, XZ is a chord of a circle. XY is the diameter of the circle, centre O. Given that</p> <ul style="list-style-type: none"> • XZ = 9 cm • YZ = $\sqrt{63}$ cm |
|---|--|



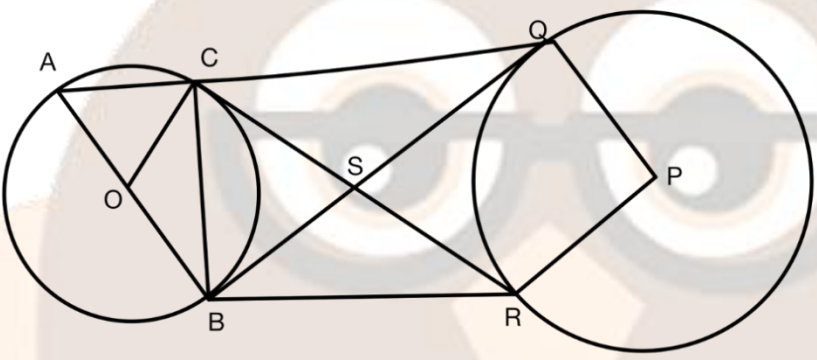
Calculate:

- (a) the length of XY
- (b) $\angle YXZ$ (in degrees)
- (c) $\angle YOZ$ (in radians)
- (d) the area of the major segment YXZ

- 5 In the diagram, line AB and line CD are tangents at points A and D respectively on the circumference of the circle with centre O.
 $\angle DAE = 33^\circ$, $\angle ECD = 59^\circ$, and AEC is a straight line.
 E, F, and G are points on the circumference of the circle.



Find

	(a) $\angle EOD$ (b) $\angle EFD$ (c) $\angle EGD$ (d) A circle is drawn with the line AC as its diameter. Explain why point D will not lie on the circumference of the circle. (e) Lines AB and CD are extended and meet at T. Show that $\angle ATD = 4^\circ$.	
6	A, B and C are points on the circle with centre O. Q and R are points on the circle with centre P. CSR and BSQ are tangents to both circles. AOB is a straight line.  (a) Show that triangle CQS is congruent to triangle BRS. Give a reason for each statement you make. (b) $\angle ABC = y^\circ$. Find, in terms of y, (i) $\angle BAC$ (ii) $\angle QPR$	

Answer Key:

1	(i) Let the radius of the enclosed circle be r cm. Since CP is a tangent to the small circle at P, $(\angle CRO = 90^\circ)$. Also, from the two tangents drawn from the external point (C) to the large circle, $\angle ROC = \frac{1.4}{2} = 0.7 \text{ radians.}$ In right-triangle ROC,
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$$\sin(\angle ROC) = \frac{RC}{CO}$$

$\Rightarrow$

$$\sin(0.7) = \frac{r}{9-r}$$

Rearrange to solve for r:

$$\sin(0.7) (9-r) = r$$

$\Rightarrow$

$$9 \sin(0.7) - r \sin(0.7) = r$$

$\Rightarrow$

$$9 \sin(0.7) = r$$

$$r = \frac{9 \sin(0.7)}{1 + \sin(0.7)}$$

$$r = \frac{9 \times 0.644217}{1 + 0.644217}$$

$$\approx \frac{5.797953}{1.644217}$$

$$\approx 3.5262 \text{ (to 4 dp)}$$

$$\approx 3.526 \text{ (to 3 dp)}$$

(ii)

$$\angle RCO = \pi - \angle CRO - \angle ROC$$

$$= \pi - \frac{\pi}{2} - 0.7$$

$$= \frac{\pi}{2} - 0.7$$

2. Angle in sector RCP .

$$\angle RCP = \pi - \angle RCO$$

$$= \pi - \left(\frac{\pi}{2} - 0.7 \right)$$

$$= \frac{\pi}{2} + 0.7$$

3. Area of sector RCP

$$\frac{1}{2} (3.526)^2 \left(0.7 - \frac{\pi}{2} \right)$$

$$\approx 14.116 \text{ cm}^2 \text{ (5 s.f.)}$$

	4. Area of triangle RCO. $= \frac{1}{2} RC \times CO \sin(\angle RCO)$ $= \frac{1}{2} (3.526) (9 - 3.526) \sin\left(\frac{n}{2} - 0.7\right).$ Since $\sin\left(\frac{n}{2} - 0.7\right) = \cos(0.7)$, Area $\approx 7.3812 \text{ cm}^2$ (5 s.f.) 5. Area of sector (OAP). Area of sector OAP $= \frac{1}{2} 9^2 (0.7)$ $= \frac{1}{2} \times 81 \times 0.7$ $= 28.35 \text{ cm}^2.$ 6. Shaded region $= 28.35 - 7.3812 - 14.116$ $\approx 6.85 \text{ cm}^2$ (3 s.f.).
2	$\angle PRS = 51^\circ$ (alternate angles) $\angle POS = 51^\circ \times 2$ (angle at centre = $2 \times$ angle at circumference) $= 102^\circ$ $\angle QPR = \angle QSR$ (angles in the same segment) $\angle PQS = \angle QSR$ (alternate angles) Since $\angle QPR = \angle PQS$, $\triangle PQT$ is an isosceles triangle $\angle$ in a semicircle is 90° OR Opposite angles in quadrilateral OPUS are supplementary. Hence points O, P, U, S can lie on the circumference of a circle. $\therefore$ Yes, they can.
3	(a) $CB = CD$ (Tangents from the same external point) Triangle BCD is isosceles.

In isosceles ($\triangle BCD$), the base angles at B and D are equal:

$$\begin{aligned}
 \angle DBC &= \angle CDB \\
 180^\circ - \angle BCD \\
 &= \frac{180^\circ - 42^\circ}{2} \\
 &= \frac{138^\circ}{2} \\
 &= 69^\circ.
 \end{aligned}$$

$$\begin{aligned}
 \angle DBP &= \angle CDB - \angle PBC \\
 &= 69^\circ - 31^\circ \\
 &= 38^\circ
 \end{aligned}$$

Therefore

$$\angle DBP = 38^\circ.$$

(b) Find $\angle RBD$.

Method 1 (using the circle's angles in the same segment and right-angles):

1. Because B, P, R, D lie on the circle, ($\angle DRP = \angle DBP$) (angles in the same segment).
Hence ($\angle DRP = 38^\circ$.)
2. PD is a diameter, so ($\angle PDR = 90^\circ$) (angle in a semicircle).
3. In ($\triangle PDR$), the third angle is

$$\begin{aligned}
 \angle RPD &= 180^\circ - \angle PDR - \angle DRP \\
 &= 180^\circ - 90^\circ - 38^\circ \\
 &= 52^\circ
 \end{aligned}$$
4. But ($\angle RPD$) is the exterior-segment angle at (D) for triangle (RBD), so ($\angle RBD = 52^\circ$.)

Method 2 (alternative via right-angles at B:

1. $\angle PBR = 90^\circ$ (angle in a semicircle on diameter PR).
2. Then $\angle RBD = 90^\circ - \angle DBP$

$$\begin{aligned}
 &= 90^\circ - 38^\circ \\
 &= 52^\circ
 \end{aligned}$$

	(c) Find ($\angle DQR$). Because P, R, D, Q also lie on the same circle, $\angle DQR$ and $\angle RPD$ are opposite angles in the circle. Thus $\begin{aligned}\angle DQR &= 180^\circ - \angle RPD \\ &= 180^\circ - 52^\circ \\ &= 128^\circ.\end{aligned}$
4	(a) By using Pythagoras' theorem in right-angled triangle XYZ, $\begin{aligned}XY &= \sqrt{XZ^2 + YZ^2} \\ &= \sqrt{9^2 + (\sqrt{63})^2} \\ &= \sqrt{81 + 63} \\ &= \sqrt{144} \\ &= 12 \text{ cm.}\end{aligned}$ (b) To find angle $\angle YXZ$, note in the right triangle that $\begin{aligned}\tan(\angle YXZ) &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{YZ}{XZ} \\ &= \frac{\sqrt{63}}{9}\end{aligned}$ Hence $\begin{aligned}\angle YXZ &= \tan^{-1} \frac{\sqrt{63}}{9} \\ &\approx 41.4^\circ \quad (\text{to 1 d.p.}).\end{aligned}$ Here is the rest of the solution in plain text. (c) $\angle YOZ$ in radians $\begin{aligned}\angle YOZ &= 2 \times \angle YXZ \\ &= 2 \times 41.4096^\circ \\ &= 82.8192^\circ.\end{aligned}$ To convert to radians, $\angle YOZ$

$$\begin{aligned}
 &= (82.8192^\circ) \times \frac{\pi}{180^\circ} \\
 &\approx 1.445469 \text{ radians} \\
 &\approx 1.45 \text{ rad (3 s.f.)}
 \end{aligned}$$

(Alternatively: in triangle OXZ,
 $(\angle XOZ = 180^\circ - 2(41.4096^\circ) = 97.1808^\circ)$,
 then $(\angle YOZ = \pi - \frac{97.1807}{180}\pi = 1.445469) \text{ rad.}$)

(d) Area of the major segment YZ

The major segment = area of the major sector YOZ plus area of triangle YOZ.

Radius $r=6$. Sector angle (major) = $(2\pi - 1.445469) \text{ rad.}$

1. Area of major sector

$$\begin{aligned}
 &\frac{1}{2} r^2 (2\pi - 1.445469) \\
 &= \frac{1}{2} (6^2) (2\pi - 1.445469).
 \end{aligned}$$

2. Area of triangle YOZ

$$\begin{aligned}
 &\frac{1}{2} r^2 \sin(\theta) \\
 &= \frac{1}{2} (6^2) \sin(1.445469).
 \end{aligned}$$

Hence total

$$\begin{aligned}
 &= \frac{1}{2} \times 36 (2\pi - 1.445469) \\
 &\quad + \frac{1}{2} \times 36 \sin(1.445469) \\
 &\approx 105 \text{ cm}^2 \quad (3 \text{ s.f.})
 \end{aligned}$$

Alternatively, using degrees:

– Major sector area

$$\begin{aligned}
 &\frac{360^\circ - 82.8192^\circ}{360^\circ} \times \pi(6)^2 \\
 &\approx 87.0789 \text{ m}^2
 \end{aligned}$$

	– Triangle area $\frac{1}{2} (6)^2 \sin(82.8192^\circ)$ $\approx 17.8168 \text{ cm}^2.$ Sum ($\approx 87.0789 + 17.8168 = 105 \text{ cm}^2$)(3s. f.)
5	(a) $\angle EOD = 66^\circ$ (angle at centre = $2 \times$ angle at circumference) (b) $\angle EFD = 33^\circ$ (angles in the same segment) (c) $\angle EGD = 147^\circ$ (angle in the opposite segment) (d) A circle is drawn with the line AC as its diameter. Explain why point D will not lie on the circumference of the circle. Answer: $\angle ADC = 180^\circ - 59^\circ - 33^\circ = 88^\circ$, and this is not 90°. Since the angle in a semicircle must be 90°, point D does not lie on the circumference. (e) Lines AB and CD are extended and meet at T. Find the angle $\angle ATD$. Answer: 1. $\angle ADC = 180^\circ - 33^\circ - 59^\circ = 88^\circ$. 2. In triangle OAD, $OA \perp AB$ (tangent $\perp$ radius), so $\angle ODA = \angle OAD = (90^\circ - 88^\circ) = 2^\circ$ (base angles of the isosceles triangle OAD). 3. $\angle BAD = 90^\circ - 2^\circ = 88^\circ$ (tangent $\perp$ radius at A). 4. In triangle ATD, sum of angles gives $\angle ATD = 180^\circ - \angle TAD - \angle ADT$ $= 180^\circ - 88^\circ - 88^\circ$ $= 4^\circ$.
6	(a) Since CSR and BSQ are tangents on both circles, applying the property of tangents from an external point, $CS = BS$ and $QS = RS$. $\angle CSQ = \angle BSR$ (vertically opposite angles) By the SAS property, $\triangle CQS \cong \triangle BRS$. (b) $\angle ABC = y^\circ$. Find, in terms of y, the following: (i) $\angle BAC$. Since $(\angle ACB = 90^\circ)$ (angle in a semicircle), and the angles of triangle ABC sum to (180°),

$$\angle BAC = 180^\circ - 90^\circ - y^\circ = (90 - y)^\circ.$$

(ii) angle QPR

We use the right angles at the tangents and vertical opposite angles:

1. ($\angle OCS = \angle OBS = 90^\circ$) (tangent $\perp$ radius).
2. ($\angle PSR = \angle PQS = 90^\circ$) (tangent $\perp$ radius).
3. ($\angle CSB = \angle RSQ$) (vertically opposite angles).

From these, quadrilateral PQRS is similar to quadrilateral OBSC.

In the isosceles triangle OBC,

$$\angle OCB = \angle OBC = y^\circ.$$

Therefore, the vertex angle at B is

$$\angle OBC + \angle OCB = y^\circ + y^\circ = 2y^\circ,$$

Hence, the remaining angle at O is

$$\angle BOC = 180^\circ - 2y^\circ.$$

By the similarity of the two quadrilaterals,

$$\angle QPR = \angle BOC = (180 - 2y)^\circ.$$

Alternative acceptable answer:

$$\angle BOC = (180 - 2y)^\circ$$