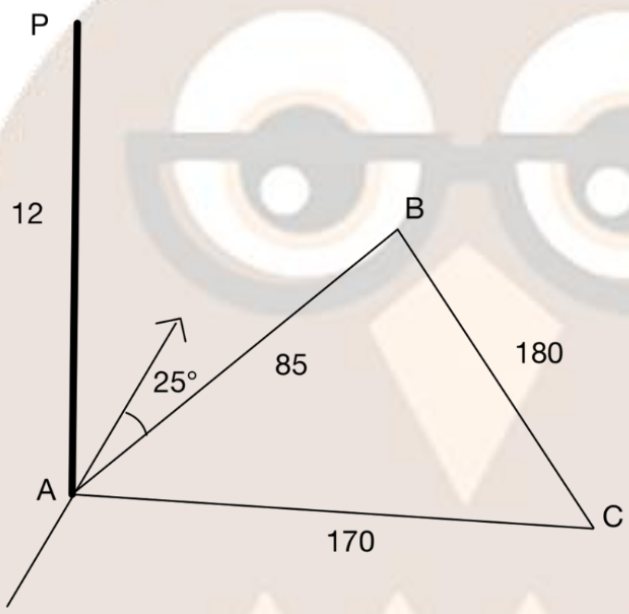
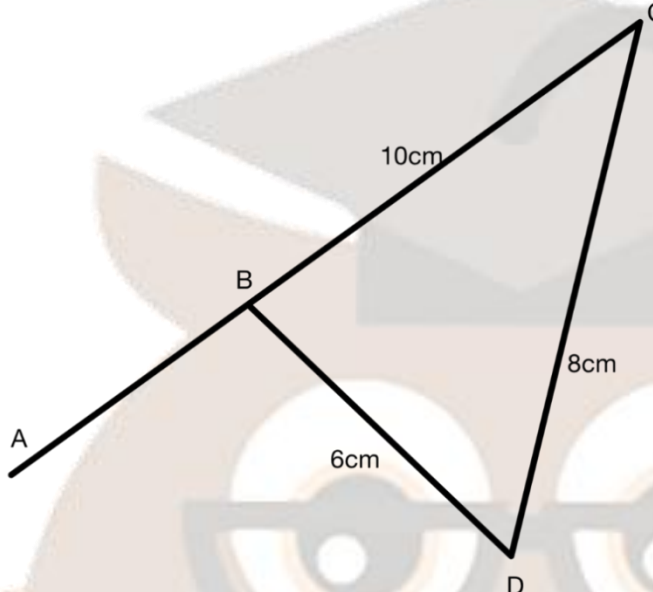
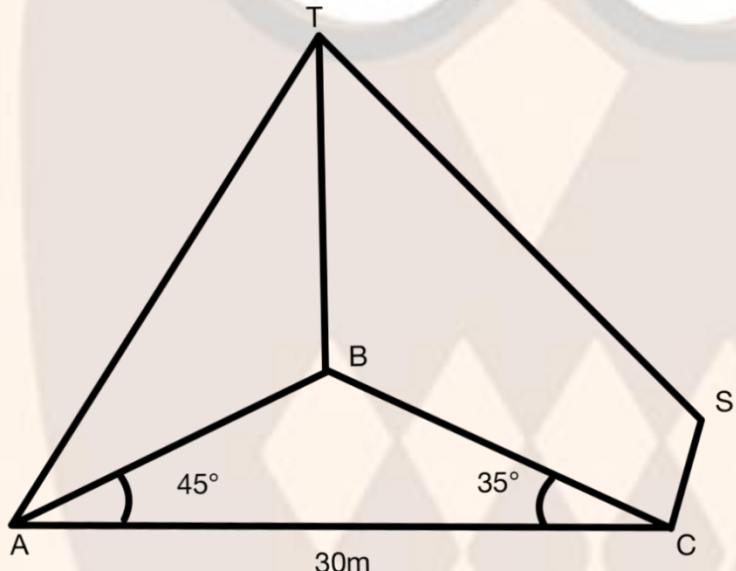


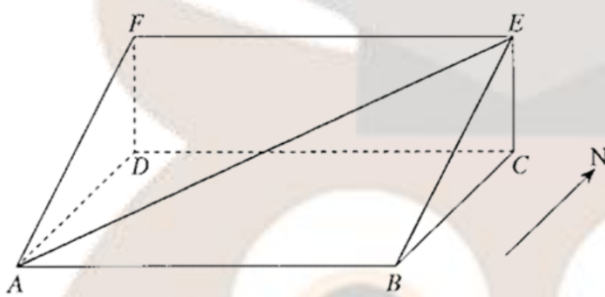
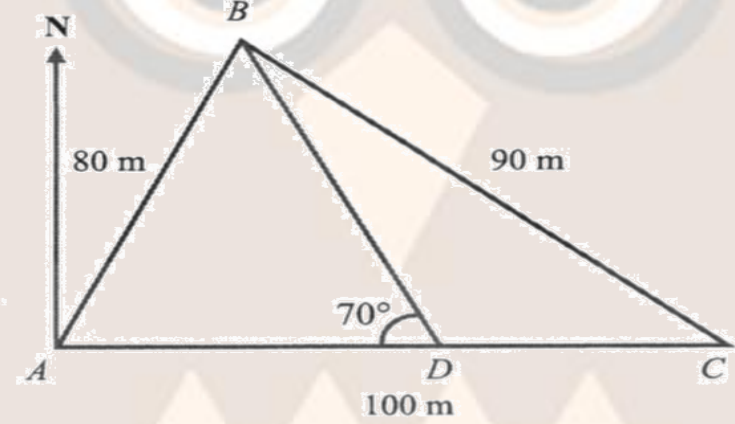
8 ESSENTIAL QUESTIONS TO MASTER

TRIGONOMETRY

1	In the diagram, ABC represents a triangular field on horizontal ground and AP represents a vertical flagpole. B is 85m from A on a bearing of 25° and C is 170m from A. The length of BC is 180m and the height of the flagpole is 12m.  (a) Calculate (i) the bearing of A from B (ii) angle ABC (iii) angle BCA	
2	In the diagram ABC is a straight line, $BC = 10$ cm, $CD = 8$ cm and $BD = 6$ cm.	

	 (a) Explain why a circle that passes through B, C, and D can be drawn in the above diagram. Give reasons for each step in your working. (b) Hence find the exact value of $\cos \angle ABD$	
3	A field is in the shape of a quadrilateral PQRS. A path crosses the field from P to R. PQ = 280 m, RS = 146 m and PR = 325 m. S is on a bearing of 42° from P, $\angle PSR = 108^\circ$ and $\angle RPQ = 38^\circ$.  (a) Calculate the bearing of R from P, giving your answer correct to the nearest degree. (b) Lin runs from Q to R at a constant speed of 5.5 km/h. Calculate the time she takes to run from Q to R. Give your answer in minutes and seconds, correct to the nearest	

	second. (c) A bird is at B, which is 25 m vertically above R. Calculate the angle of depression of S from B.	
4	On a map whose scale is 8 cm to 1 km, an estate is shown as a quadrilateral ABCD. The length of the diagonal AC is 7 cm, $\angle BAC = 55^\circ$, $\angle BCA = 77^\circ$, $\angle DAC = 90^\circ$ and $\angle DCA = 40^\circ$. Calculate: (a) the length, in cm, of the side AB on the map (b) the length, in km, which is represented by AD (c) the area, in km^2, which is represented by $\triangle ADC$.	
5	A man on a ship observes that the angle of elevation of a lighthouse is 12° . When he sails a further distance of 200 m away from the lighthouse, the angle of elevation becomes 10° . Find the height of the lighthouse.	
6	A, B and C are three points on horizontal ground. It is given that $AC = 30$ m, $\angle BAC = 45^\circ$ and $\angle BCA = 35^\circ$. T is the top of a vertical tower and S is the top of a vertical statue standing at C. The angle of elevation of T from A is 58°.  (a) Find the (i) distance AB, (ii) height of the tower. (b) If the angle of depression of S from T is 49°, find the height of the statue.	
7	The diagram shows a hill slope ABEF facing the north. Point E is directly above point C and point F is directly above point D. there are two paths on the slope, AE and BE. The bearing of C from A is 67°. ABCD and ABEF are rectangles. $BE = 65$ m and $\angle CBE = 21^\circ$ (a) find the length of (i) BC	

	(ii) AC. (b) If M is the midpoint of AC, find the bearing of M from B. (c) There is a tree located at point E. The angle of elevation of the tip of tree from point A is 52°. Calculate the height of the tree (d) Which path is steeper, AE or BE? Explain your answer by showing your calculations. 	
8	The diagram below shows three points, A, B and C on flat ground. $AB = 80$ m, $AC = 100$ m and $BC = 90$ m. C is due east of A.  (a) (i) Show that angle $ACB = 49.5^\circ$, correct to one decimal place. (ii) Hence, calculate the bearing of C from B. (iii) Point D lies along AC such that angle $BDA = 70^\circ$. Find the distance CD. (iv) Find the ratio of area of triangle ABD to area of triangle BDC. Give your answer correct to one decimal place. (b) The diagram below shows a cuboid with rectangular base ABCD. $AB = 4$ cm, $BC = 3$ cm and $AE = 12$ cm. M is the midpoint of AE. Find the angle of elevation of G from M.	

Answer Key:

1	(a) $180 - 25 = 155$ Bearing = $360 - 155 = 205^\circ$ (b) $\cos^{-1}\left(\frac{170^2 - 85^2 - 180^2}{-2(85)(180)}\right)$ $= 69.4827$ $= 69.5^\circ$ (1 d.p.) (c) $\frac{\sin 69.4827}{170} = \frac{\sin C}{85}$ (accept cosine rule as well) $\angle BCA = 27.9^\circ$	
2	(a) $BC^2 = 100$ $BD^2 + CD^2 = 64 + 36 = 100$ $BD^2 + CD^2 = BC^2$ By the converse of Pythagoras' theorem, angle BDC is 90°. A circle that passes through B, C, and D can be drawn (angle in a semicircle). (b) $\frac{3}{5}$	
3	(a) $\frac{\sin SPR}{146} = \frac{\sin 108}{325}$ $\sin SPR = \frac{\sin 108}{325} \times 146$ $SPR = 25.29$ $25.29 + 42 = 67.29$ Bearing = 67° (b) $QR^2 = 280^2 + 325^2 - 2 \times 280 \times 325 \times \cos 38$ $QR = 201.51$ $\frac{201.51}{1000 \times 5.5} \times 60 \times 60$ 2 min 12 sec (c) $\tan^{-1}\left(\frac{25}{146}\right)$ $= 9.7^\circ$	
4	(a) To find the length of AB, we can use the Law of Sines in $\triangle ABC$. First, we find $\angle ABC$: $\angle ABC = 180^\circ - \angle BAC - \angle BCA = 180^\circ - 55^\circ - 77^\circ = 48^\circ$ Now, using the Law of Sines:	

	$\frac{AB}{\sin \angle BCA} = \frac{AC}{\sin \angle ABC}$ $AB = \frac{AC \times \sin \angle BCA}{\sin \angle ABC} = \frac{7 \times \sin 77^\circ}{\sin 48^\circ}$ $AB \approx \frac{7 \times 0.9744}{0.7431} \approx 9.17 \text{ cm}$ (b) In $\triangle ADC$, $\angle DAC = 90^\circ$ and $\angle DCA = 40^\circ$. We can use trigonometric ratios to find AD. $\tan \angle DCA = \frac{AD}{AC}$ $AD = AC \times \tan \angle DCA = 7 \times \tan 40^\circ$ $AD \approx 7 \times 0.8391 \approx 5.87 \text{ cm}$ Since 8 cm on the map represents 1 km in reality, we can find the length represented by AD in km: $AD_{\text{km}} = \frac{5.87}{8} \approx 0.73 \text{ km}$ (c) The area of $\triangle ADC$ can be calculated using the formula: $\text{Area} = \frac{1}{2} \times AC \times AD = \frac{1}{2} \times 7 \times 5.87 \approx 20.545 \text{ cm}^2$ Now, we need to convert this area to km^2. Since 8 cm = 1 km, then 1 cm = $\frac{1}{8}$ km. Therefore, $1 \text{ cm}^2 = \left(\frac{1}{8}\right)^2 \text{ km}^2 = \frac{1}{64} \text{ km}^2$. $\text{Area}_{\text{km}^2} = 20.545 \times \frac{1}{64} \approx 0.32 \text{ km}^2$	
5	Let h be the height of the lighthouse. Let x be the initial distance from the ship to the lighthouse. We can set up two equations using the tangent function: 1. From the initial position: $\tan(12^\circ) = \frac{h}{x}$ 2. After sailing 200 m away: $\tan(10^\circ) = \frac{h}{x+200}$ We have a system of two equations with two unknowns, h and x. We can solve for h. First, solve each equation for x: $x = \frac{h}{\tan(12^\circ)}$ $x + 200 = \frac{h}{\tan(10^\circ)}$ Substitute the first equation into the second equation:	

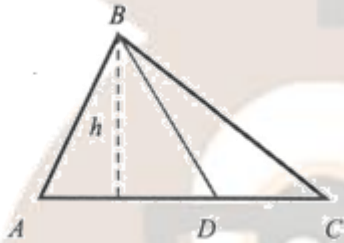
	$\frac{h}{\tan(12^\circ)} + 200 = \frac{h}{\tan(10^\circ)}$ Now, solve for h: $200 = \frac{h}{\tan(10^\circ)} - \frac{h}{\tan(12^\circ)}$ $200 = h \left(\frac{1}{\tan(10^\circ)} - \frac{1}{\tan(12^\circ)} \right)$ $h = \frac{200}{\frac{1}{\tan(10^\circ)} - \frac{1}{\tan(12^\circ)}}$ Using a calculator: $\tan(10^\circ) \approx 0.1763$ $\tan(12^\circ) \approx 0.2126$ $h = \frac{200}{\frac{1}{0.1763} - \frac{1}{0.2126}}$ $h = \frac{200}{5.6728 - 4.7035}$ $h = \frac{200}{0.9693}$ $h \approx 206.33 \text{ m}$ Therefore, the height of the lighthouse is approximately 206.33 meters.	
6	(a) (i) To find $\angle ABC$: $\angle ABC = 180^\circ - 45^\circ - 35^\circ = 100^\circ \quad (\text{angle sum of a triangle})$ Using the Sine Rule: $\frac{AB}{\sin 35^\circ} = \frac{30}{\sin 100^\circ}$ $AB = \frac{30 \times \sin 35^\circ}{\sin 100^\circ} \approx 17.473 \approx 17.5 \text{ m (3 s.f.)}$ (ii) $\tan 58^\circ = \frac{TB}{17.473}$ $TB = 27.962$ Height of tower is: 28.0 m (3 s.f.) (b)	

	 Using the Sine Rule: $\frac{BC}{\sin 45^\circ} = \frac{30}{\sin 100^\circ}$ $BC = 21.54 \text{ m (4 s.f.)}$ $RS = BC = 21.54 \text{ m}$ $\angle TSR = 49^\circ \text{ (alt. } \angle \text{s)}$ $\tan 49^\circ = \frac{TR}{21.54}$ $TR = 24.779 \text{ m (5 s.f.)}$ $\therefore \text{Height of the statue} = 28.0 - 24.779 = 3.22 \text{ m (3 s.f.)}$	
7	(a) (i) $\cos 21^\circ = \frac{BC}{65}$ $BC = 60.68273$ $\approx 60.7 \text{ m (3 s.f.)}$ (ii) $\sin 23^\circ = \frac{BC}{AC}$ $AC = \frac{60.68273}{\sin 23^\circ}$ $= 155.3056$ $\approx 155 \text{ m (3 s.f.)}$ (b) $\angle MBA = 23^\circ$ Bearing of M from B $= 270^\circ + 23^\circ$ $= 293^\circ$ (c)	

	$\sin 21^\circ = \frac{EC}{65}$ $EC = 23.294 \text{ m}$ Let the height of the tree be h m. $\tan 52^\circ = \frac{h + 23.294}{155.3056}$ $h + 23.294 = 198.782$ $h = 175.488$ Therefore, height of the tree is 175 m (3 s.f.). (d) Gradient of path BE $= \tan \angle EBC$ $= \tan 21^\circ$ $= 0.3839 \text{ (4 d.p.)}$ Gradient of path AE $= \tan \angle EAC$ $= \frac{EC}{AE}$ $= \frac{23.294}{155.3056}$ $= 0.1500 \text{ (4 d.p.)}$ Since gradient of path BE > gradient of path AE, BE is steeper.	
8	(a) (i) $\cos \angle ACB = \frac{90^2 + 100^2 - 80^2}{2(90)(100)}$ $\angle ACB = \cos^{-1} \left(\frac{90^2 + 100^2 - 80^2}{2(90)(100)} \right)$ $= 49.458^\circ$ $= 49.5^\circ (\text{shown})$ (ii) $\text{Bearing of } C \text{ from } B = 90^\circ + 49.458^\circ$ $= 139.5^\circ \text{ (1 dec. place)}$ (iii) $\angle BDC = 180^\circ - 70^\circ (\text{Adjacent angles on a straight line})$ $= 110^\circ$ $\angle CBD = 180^\circ - 110^\circ - 49.458^\circ$ $= 20.542^\circ [\text{M1 - Finding } \angle CBD]$	

$$\begin{aligned}
 \frac{90}{\sin 110^\circ} &= \frac{CD}{\sin 20.542^\circ} \quad [M1 - \text{Sine Rule}] \\
 CD &= \frac{90 \sin 20.542^\circ}{\sin 110^\circ} \\
 &= 33.606 \\
 &= 33.6 \text{ m (3 sig. fig.)}
 \end{aligned}$$

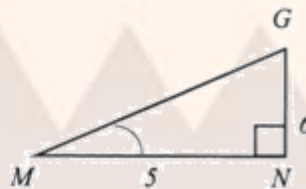
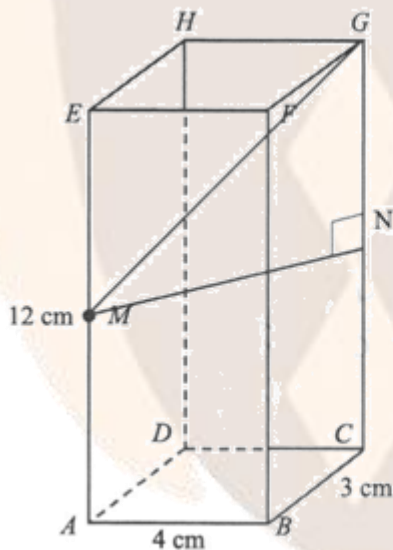
(iv)



Since they share a common height, $\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle BDC} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(DC)(h)}$ [M1 –
 Compare ratio with common height]

$$\begin{aligned}
 &= \frac{AD}{CD} \\
 &= \frac{100 - 33.606}{33.606} \\
 &= 1.975 \\
 &-2.0 \text{ (1 dec. place)} \quad [A11 - \text{Accept } 2.0: 1.0]
 \end{aligned}$$

(b)



	Let N be the midpoint of GC. $AC = \sqrt{4^2 + 3^2}$ $= 5\text{cm} \quad [M1 - \text{Finding } AC]$ $GN = \frac{12}{2} = 6\text{cm}$ $\tan \angle GMN = \frac{GN}{MN}$ $= \frac{6}{5} \quad [M1 - \text{Using tangent}]$ $\angle GMN = \tan^{-1} \left(\frac{6}{5} \right)$ $= 50.194^\circ$ $= 50.2^\circ \text{ (1 dec. place)}$	
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