

8 ESSENTIAL QUESTIONS TO MASTER

PROPORTIONS

1	y is directly proportional to the cube of x. Given that $y = 4$ when $x = 2$, find (a) an equation connecting y and x. (b) Find the value of x when $y = 10$ (c) When x is doubled, y is eight times its original value. Prove that the above statement is correct.	
2	It is given that y^2 is inversely proportional to x and $y = 4$ when $x = \frac{1}{2}$. (a) Find the equation connecting y and x. (b) Find the value of x when $y = 6$.	
3	3 people can fold 360 pieces of t-shirts in 3 hours. (a) How long does it take 1 person to fold 480 pieces of t-shirts? (b) State the assumption that you have made for your answer in part (a).	
4	Given that y is directly proportional to the cube of $(x + 1)$ and that $y = 15$ when $x = 2$. (a) Express y in terms of x. (b) Find the value of x when $y = 120$. (c) Find the value of y when $x = 11$.	
5	(a) A group of men took 10 days to build a structure. Assuming the men work at the same rate, find the number of days to build the structure if the number of men is doubled. (b) A builder is to build a rectangular playground with dimensions x m by y m. Given that x and y are inversely proportional, and $y = 34$ when $x = 20$. (i) Find the equation connecting x and y. (ii) Find the value of y when $x = 15$.	
6	y is connected to x by the equation $y = \frac{5}{2x}$. (a) Find the value y when $x = 5$. (b) y is directly proportional to x. Do you agree with the above statement? Explain your answer.	
7	The number of hours, N, required to dig a trench, is inversely proportional to the square of the number of men available, X. It takes 6 men a total of 8 hours to dig a trench. (a) Find an equation connecting N and X. (b) When X is increased by 100%, find the percentage reduction in the value of N.	
8	(a) z is inversely proportional to the cube of w. Given that $z = 16$ for a particular value of w, find the value of z when the value of w is tripled. (b) m is directly proportional to n and inversely proportional to the square of r. Given	

	that $m = 10$ when $n = 5$ and $r = 2$, find the value of m when n is doubled and r is halved.	
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Answer Key:

1	(a) $4 = k(2)^3$ $4 = k(8)$ $k = \frac{4}{8} = \frac{1}{2}$ $y = \frac{1}{2}x^3$ (b) $10 = \frac{1}{2}x^3$ $x^3 = 20$ $x = 2.71$ (3 s.f.) (c) $y_{original} = \frac{1}{2}x^3$ When x is doubled, New $y = \frac{1}{2}(2x)^3$ $= \frac{1}{2}(8x^3)$ $= 8\left(\frac{1}{2}x^3\right)$ $= 8(y_{original})$ Or When $x = 4$, $y = \frac{1}{2}(4)^3 = \frac{1}{2}(64) = 32$ $\frac{y_{new}}{y_{original}} = \frac{32}{4} = 8$ Do not accept: $\frac{1}{8}$	
2	(a) $u^2 = \frac{k}{x}$, where k is a constant. $y^2 = \frac{k}{x}$ $4^2 = \frac{k}{2}$ $k = 32$ $k = \frac{16}{2} = 8$ $y^2 = \frac{k}{x}$	

	(b) You have the equation $(6)^2 = \frac{8}{x}$. You want to solve for x, and you have $x = \frac{8}{36}$. To simplify the fraction, we can divide both the numerator and the denominator by their greatest common divisor, which is 4. $x = \frac{8 \div 4}{36 \div 4} = \frac{2}{9}$ Therefore, $x = \frac{2}{9}$.	
3	(a) 3 people complete 360 units of work in 3 hours. 1 person completes 120 units of work in 1 hour. 1 person completes 480 units of work in 12 hours (b) Assume that all of the people have the same folding speed.	
4	(a) The equation is of the form $y = k(x + 1)^3$. Given the point (2,15), we substitute $x = 2$ and $y = 15$ into the equation to solve for k: $15 = k(2 + 1)^3$ $15 = k(3)^3$ $15 = k(27)$ Divide both sides by 27 to isolate k: $k = \frac{15}{27} = \frac{5}{9}$ Substitute the value of k back into the original equation: $y = \frac{5}{9}(x + 1)^3$ (b) $y = 120$ $120 = \frac{5}{9}(x + 1)^3$ $216 = (x + 1)^3$ $6 = x + 1$ $x = 5$ (c) Given: $x = 11$ Equation: $y = \frac{5}{9}(11 + 1)^3$ Simplifying:	

	$y = \frac{5}{9}(12)^3$ Solution: $y = 960$	
5	(a) Number of days = 5 (b)(i) $k = 34 \times 20$ $k = 680$ $x = \frac{680}{y}$ or $y = \frac{680}{x}$ (b)(ii) $y = \frac{680}{15} = 45\frac{1}{3}$	
6	(a) $y = \frac{1}{2}$ (b) No, as y decreases when x increases, y is not directly proportional to x . OR No, as y decreases when x increases, y is inversely proportional to x .	
7	(a) $N = \frac{k}{X^2}$, where k is a constant and $k \neq 0$ When $N = 8, X = 6$, $8 = \frac{k}{6^2}$ $k = 8 \times 6^2$ $k = 288$ $\therefore N = \frac{288}{X^2}$ or $NX^2 = 288$ (b) $X_{new} = 2X$ Percentage change in N: $\frac{\frac{288}{(2X)^2} - \frac{288}{X^2}}{\frac{288}{X^2}} \times 100\%$ $= \frac{\frac{288}{4X^2} - \frac{288}{X^2}}{\frac{288}{X^2}} \times 100\%$ $= -75\%$ Therefore, the percentage reduction is 75%.	
8	(a) Since z is inversely proportional to the cube of w , we can write the relationship as: $z = \frac{k}{w^3}$ where k is the constant of proportionality. Given $z = 16$ for a particular value of w , let's call that w_1 . Then:	

$$16 = \frac{k}{w_1^3}$$

$$\text{So, } k = 16w_1^3.$$

Now, we want to find the value of z when w is tripled, i.e., $w = 3w_1$. Let's call this new value of z , z_2 .

$$z_2 = \frac{k}{(3w_1)^3} = \frac{16w_1^3}{27w_1^3} = \frac{16}{27}$$

So, the value of z when w is tripled is $\frac{16}{27}$.

(b) Since m is directly proportional to n and inversely proportional to the square of r , we can write the relationship as:

$$m = \frac{kn}{r^2}$$

where k is the constant of proportionality.

Given $m = 10$ when $n = 5$ and $r = 2$, we can find k :

$$10 = \frac{k \times 5}{2^2} = \frac{5k}{4}$$

$$\text{So, } k = \frac{10 \times 4}{5} = 8.$$

Now, we want to find the value of m when n is doubled (i.e., $n = 2 \times 5 = 10$) and r is halved (i.e., $r = \frac{2}{2} = 1$). Let's call this new value of m , m_2 .

$$m_2 = \frac{8 \times 10}{1^2} = \frac{80}{1} = 80$$

So, the value of m when n is doubled and r is halved is 80.