

8 ESSENTIAL QUESTIONS TO MASTER

NUMBER PATTERNS

1 A series of diagrams consisting of black and white triangles is shown below. The black triangles line the outer perimeter of each figure. All the other small triangles are white.



Fig. 1



Fig. 2



Fig. 3



Fig. 4

The table shows the number of small triangles.

Figure	1	2	3	4	5	...	n
No of Black Triangles	3	6	9	12	c	...	x
No of White Triangles	1	3	7	13	d	...	y
Total number of Triangles	4	9	16	25	e	...	z

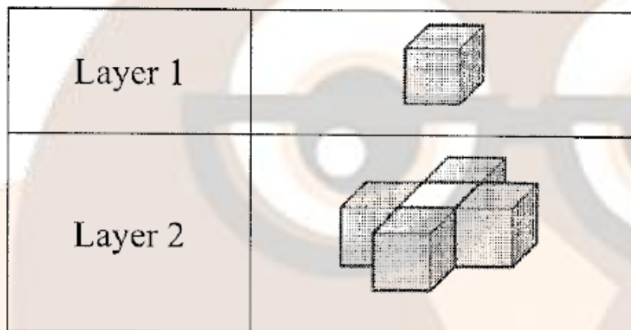
(a) (i) State the value of c.
 (ii) Write down an expression, in terms of n, for x.
 (b) (i) State the value of e.
 (ii) Write down an expression, in terms of n, for z.
 (c) Write down the relationship between x, y and z in the form of an equation.
 (d) How many white triangles are there in figure 73?
 (e) Is it possible for any figure to have a total number of 1849 triangles? Explain your answer.

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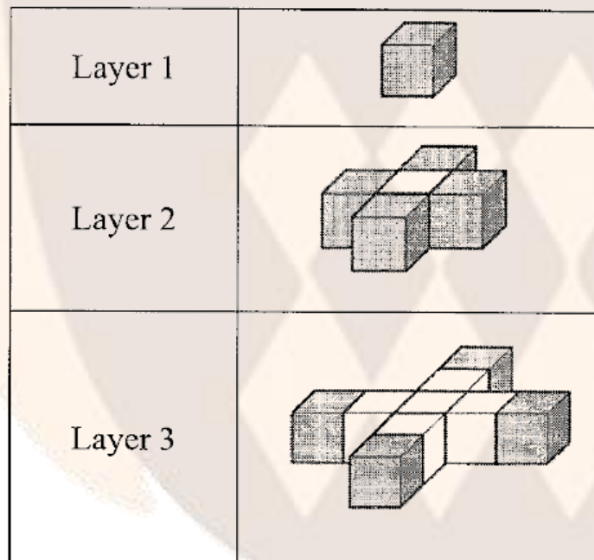
A tower is made of grey and white cubes.
The first layer of the tower is made with one grey cube.



A tower with two layers is made with five grey cubes and one white cube as shown below.



A tower with three layers is made with nine grey cubes and six white cubes as shown below.



	(a) Complete the table below.					
	Number of Layers	1	2	3	4	5
	Total number of white cubes	0	1	6		
	Total number of grey cubes	1	5	9		
	Total number of cubes	1	6	15		
	(b) Find, in terms of n , the total number of grey cubes in a tower with n layers.					
	(c) Find, in terms of n , the total number of cubes in a tower with n layers.					
	(d) Sarah used 97 grey cubes to build a tower.					
	Calculate the number of white cubes she used.					
3	The first four diagrams in a sequence are shown below.					
						
	Diagram 1 3 dots	Diagram 2 10 dots	Diagram 3 21 dots	Diagram 4 36 dots		
	(a) Find the number of dots in Diagram 5.					
	(b) Find an expression, in terms of n , for the number of dots in Diagram n .					
	(c) Find the difference in the number of dots between Diagram 7 and Diagram 6.					
	(d) Show that the difference in the number of dots between Diagram $(n + 1)$ and Diagram n is $4n + 3$.					

4	The table shows a sequence of figures formed by drawing lines. <table><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td>Figure 1</td><td>Figure 2</td><td>Figure 3</td></tr><tr><td>Number of non-vertical lines</td><td>2</td><td>6</td><td>12</td></tr></table> (a) Find the number of non-vertical lines in figure 4 and figure 5. (b) Find an expression, in terms of n, in the form $an^2 + bn$ for the number of non-vertical lines found in figure n. (c) Find the number of non-vertical lines in figure 56. (d) Hadi claims that one of the figures has 243 non-vertical lines. Is he correct? Explain your answer.						Figure 1	Figure 2	Figure 3	Number of non-vertical lines	2	6	12	
														
	Figure 1	Figure 2	Figure 3											
Number of non-vertical lines	2	6	12											
5	The first three terms in a sequence of numbers, $(T_1, T_2, T_3, \dots)$ are given below. $T_1 = 1 - \frac{1}{2}$ $T_2 = \frac{1}{2} - \frac{1}{3}$ $T_3 = \frac{1}{3} - \frac{1}{4}$ (a) Write down T_4. (b) Show that the total sum of $T_1, T_2, T_3, \dots, T_n$ in the above sequence is $1 - \frac{1}{n+1}$.													
6	A L-shaped block is made of 3 squares of $1 - \text{cm}^2$ each and has a perimeter of 8 cm. The following pattern shows L-shaped blocks joined together.  1 L-shaped block 2 L-shaped blocks 3 L-shaped blocks The values for the perimeter of the 1 L-shaped block and 2 L-shaped blocks are given below. (i) Find a and b. (ii) Write down the perimeter of a n L-shaped block in terms of n.													

	(iii) If the perimeter of an n L-shaped block is 2258 cm, find the number of squares in this n L-shaped block.	
7	Consider the number pattern below. $\frac{1}{11} = 0.090909 \dots = 0.\dot{0}\dot{9}$ $\frac{2}{11} = 0.181818 \dots = 0.\dot{1}\dot{8}$ $\frac{3}{11} = 0.272727 \dots = 0.\dot{2}\dot{7}$ $\frac{4}{11} = 0.363636 \dots = 0.\dot{3}\dot{6}$ $\vdots$ $\frac{x}{11} = 636.3636 \dots = 636.\dot{3}\dot{6}$ (a) Write down the 5 th line of the pattern. (b) Find the value of x .	
8	Consider the number pattern below. $\frac{1}{13} = 0.076923076923\dots = 0.\overline{076923}$ $\frac{2}{13} = 0.153846153846\dots = 0.\overline{153846}$ $\frac{3}{13} = 0.230769230769\dots = 0.\overline{230769}$ $\frac{4}{13} = 0.307692307692\dots = 0.\overline{307692}$ $\frac{y}{13} = 807.6923076923\dots = 807.\overline{692307}$ (a) Write down the 5 th line of the pattern. (b) Find the value of y .	

Answer Key:

1	(a) (i) 15 (ii) $3n$ (b) (i) 36 (ii) $(n + 1)^2$ or $n^2 + 2n + 1$ (c) $y = z - x$ or $y = (n + 1)^2 - 3n$ (d) $(73 + 1)^2 - 3(73) = 5257$ (e) Yes $\sqrt{1849} = 43$ 1849 is a perfect square or the 42 th term has 1849 triangles.	

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(a)

Number of Layers	1	2	3	4	5	n
Total number of white cubes	0	1	6	15	28	$(n-1)(2n-3)$
Total number of grey cubes	1	5	9	13	17	$(4n-3)$
Total number of cubes	1	6	15	28	45	$(2n^2 - n)$

(b)

 For $n = 1$, number of grey cubes = 1

 For $n = 2$, number of grey cubes = $5 = 1 + 4$

 For $n = 3$, number of grey cubes = $9 = 1 + 4 + 4$

 For $n = 4$, number of grey cubes = $13 = 1 + 4 + 4 + 4$

 For $n = 5$, number of grey cubes = $17 = 1 + 4 + 4 + 4 + 4 + 4$

 Total number of grey cubes in a tower with n layers

$$= 1 + (n-1)(4)$$

$$= 1 + 4n - 4$$

or

$$= 4n - 3$$

(c)

 For $n=1$, total no. cubes = $1 = 1 \times 1$

 For $n=2$, total no. cubes = $6 = 2 \times 3$

 For $n=3$, total no. cubes = $15 = 3 \times 5$

 For $n=4$, total no. cubes = $28 = 4 \times 7$

 For $n=5$, total no. cubes = $45 = 5 \times 9$

 Total no of grey cubes in a tower with n layers

$$= n \times (2n - 1)$$

or

$$= 2n^2 - n$$

(d)

$$4n - 3 = 97$$

$$4n = 100$$

	$n = 25$ Total no of cubes to build a tower with 25 layers $= 25(50 - 1)$ $= 25(49)$ $= 1225$ No. of white cubes used $= 1225 - 97$ $= 1128$	
3	(a) 55 (b) $n(2n+1)$ (c) 27 (d) show $(n + 1)(2(n + 1) + 1) - n(2n + 1)$ leading to $4n+3$	
4	(a) Figure 4 = $4 \times 5 = 20$ Figure 5 = $5 \times 6 = 30$ (b) For Figure 1, the expression is $1^2 + 1 = 2$. For Figure 2, the expression is $2^2 + 2 = 6$. For Figure 3, the expression is $3^2 + 3 = 12$. The expression for the number of non-vertical lines in Figure n is given by $n^2 + n$. This formula represents the pattern observed in the sequence. (c) The number of non-vertical lines in figure 56 is calculated as $56^2 + 56 = 3192$. (d) <u>Method 1</u> According to Hadi's claim, $n^2 + n = 243$ $n^2 + n - 243 = 0$ $n = \frac{-1 \pm \sqrt{973}}{2} \approx 15.0964$ n represents the Figure Number and thus, it must be a positive integer. Since n is not a positive integer, Hadi's claim is wrong as there is no figure with 243 non-vertical lines. <u>Method 2</u> Number of non-vertical lines $= n^2 + n$	

	$= n(n + 1)$ n represents the Figure Number and thus, it must be a positive integer. Since $n(n+1)$ represents the product of two consecutive integers and it will always result in an even integer, Hadi's claim is wrong as there is no figure with 243 non-vertical lines as it is an odd integer. <u>Method 3</u> Fig 15 = 240 Fig 16 = 272 Since n represents the Figure Number and it must be a positive integer, there is no figure with 243 non-vertical lines.	
5	(a) The equation is $T_4 = \frac{1}{4} - \frac{1}{5}$. (b) $\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$	
6	(i) $a = 20, b = 26$ (ii) $6n + 2$ (iii) $6n + 2 = 2258$ $n = 376$ No. of squares = 1128	
7	(a) $\frac{5}{11} = 0.454545 \dots = 0.\overline{45}$ (b) $\frac{x}{11} = 636.3636 = 636 + 0.3636$ $\frac{x}{11} = 636 + \frac{4}{11}$ $x = 11(636) + 4$ $x = 7000$	
8	(a) Write down the line of the pattern Following the pattern, the next numerator is 5: $\frac{5}{13} = 0.384615384615 \dots = 0.\overline{384615}$ So, the 5th line is:	

	$\frac{5}{13} = 0.384615384615 \dots = 0.\overline{384615}$ (b) Find the value of given First, express $807.\overline{692307}$ as a fraction. Let $= 807.\overline{692307}$. The repeating part has 6 digits. Let's write: $n = 807.692307692307 \dots$ Multiply both sides by $10^6 = 1,000,000$ to shift the decimal point 6 places: $1,000,000n = 807692307.692307692307 \dots - 807.692307692307 \dots$ $999,999n =$ $807692307 - 807 = 691500$ $999,999n = 807691500$ $n = \frac{807691500}{999,999}$	
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