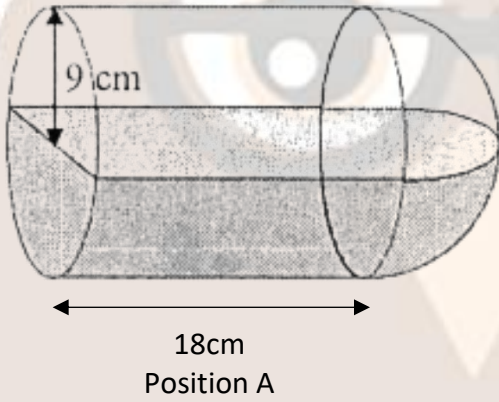
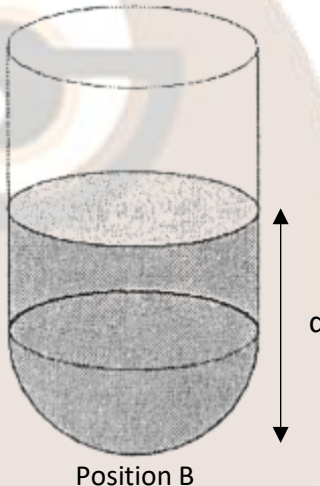
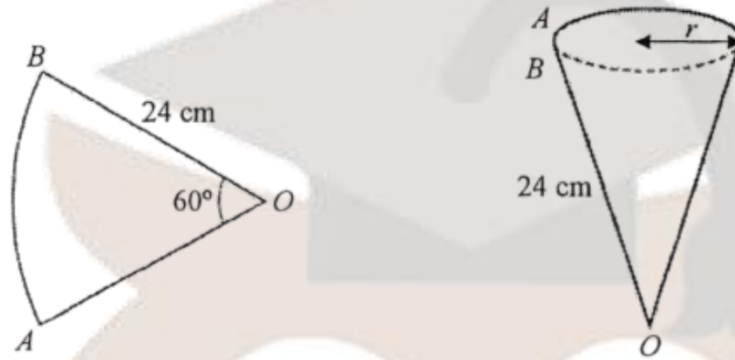
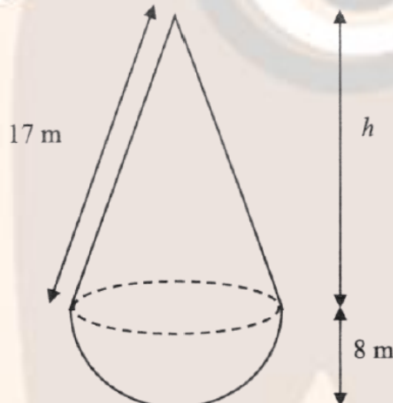
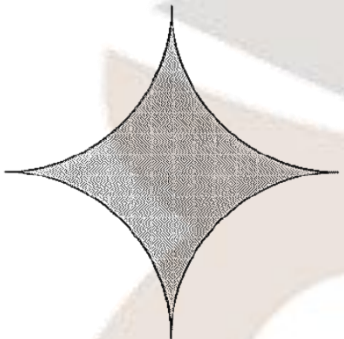
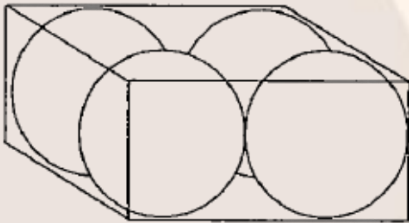
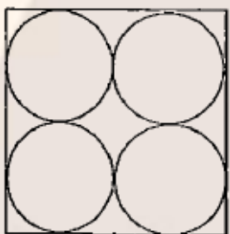


7 ESSENTIAL QUESTIONS TO MASTER

MEASURATION

1	A closed container, made by joining a hemisphere of radius 9 cm and a cylinder of length 18 cm, is placed in two rest positions as shown below. In Position A, the container is horizontal and exactly half filled with water. In Position B, the half-filled container is held with its axis vertical and the hemisphere at the bottom.   (i) Find the volume of water in the container, leaving your answer in pi. (ii) Show that the depth of the water, d, in Position B is 15 cm. (iii) Find the total surface area of the container in contact with the water in Position B.	
2	The wrapper of a giant ice-cream cone is formed from joining points A and B of the sector AOB. (a) Find the area of sector AOB in terms of n. (b) Show that the radius, r, of the ice-cream cone wrapper is 4 cm. (c) Find the volume of the ice-cream cone.	

		
3	The diagram below shows a solid made from a cone and a hemisphere. (a) Show that the height, h, of the cone is 15 m. (b) Calculate the total surface area of the solid.  (c) The solid is melted to form geometrically similar solids, to be used as lighting ornaments. The height of the ornament should not exceed 0.1 m. Suggest the number of such ornaments to be produced. Show your workings clearly. 	

4	Four identical quadrants are cut from the corners of a square, leaving the remaining shape as shown below. If the length of the arc of each quadrant is 3π cm, find the area of the remaining shape. Give your answer in terms of π. 	
5	Fig 7.1 (not drawn to scale) shows 4 identical spherical iron balls packed into a transparent box. Fig 7.2 shows the top view of the box. Each ball is in contact with the interior surfaces of the box and the balls beside it. It is given that the radius of each ball is 3 cm.  Fig 7.1  Fig 7.2 Top View (a) Calculate - the total volume of the four iron balls, in terms of π, - the volume of the box, - the percentage of the space in the box that is not occupied by the balls. (b) The 4 iron balls are melted and moulded into a solid cone as shown in Fig 7.3. The cone has a height of 12 cm and its base radius is x cm.	

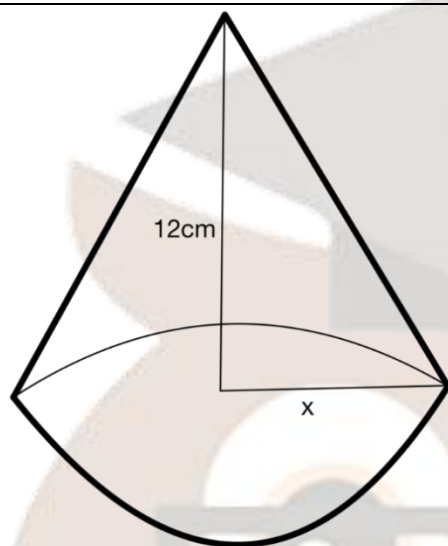
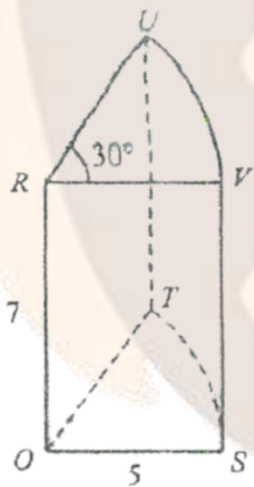
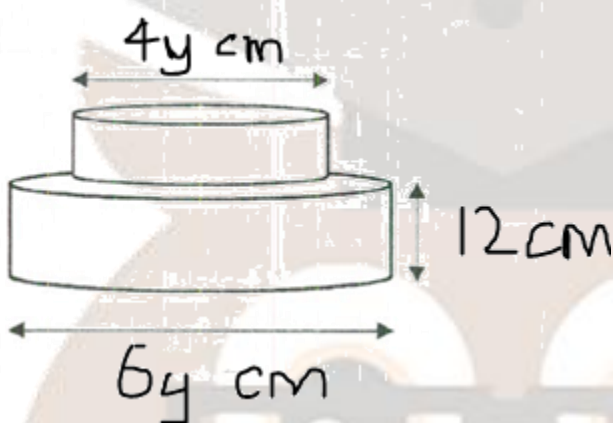


Figure 7.3

- (i) Using your answer from (a)(i), show that the radius, x , is 6cm .
 (ii) Calculate the total surface area of the cone.

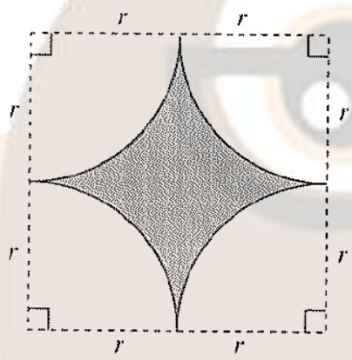
- 6 The figure above shows a solid. The cross-section of the solid is a sector of a circle of radius 5 cm and angle 30° . The horizontal cross-sections, OST and RVU are 7 cm apart. S , T , Y and V lie on the curved surface of the solid. The line OR , TU and SV are vertical. Find
- (i) The area of the curved surface $STUV$ in terms of π
 (ii) The angle UST
 (iii) Another solid geometrically similar to the given figure has a base radius of 3 cm . Find the ratio of the volume of the smaller solid to the volume of the larger one.



7	The diagram shows a birthday cake made up of 2 geometrically similar cylindrical tiers. The diameters of the upper tier and lower tier are 4y cm and 6y cm respectively. The height of the lower tier is 12 cm.  (a) Find the total height of the cake. (b) Given that the cake weighs a total of 2 kg, find the weight of the lower tier.	
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1	(a) (i) Volume of cylinder $= \pi(9)^2(18)$ $= 1458\pi$ Volume of hemisphere $= \frac{2}{3}\pi(9)^3$ $= 486\pi$ Volume of water $= \frac{1}{2}(1458\pi + 486\pi)$ $= 972\pi \text{ cm}^3$ (a)(ii) Volume of cylinder $= 972\pi - 486\pi$ $= 486\pi \text{ cm}^3$ $\pi(9)^2 h = 486$ $h = 6 \text{ cm}$ $d = 6 + 9 = 15 \text{ cm (shown)}$	
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	(a)(iii) Total surface area in contact with water: $2\pi(9)^2 + 2\pi(9)(6) = 848.23 \approx 848 \text{ cm}^2$	
2	(a) Area of sector (AOB) $= \frac{1}{2}(24)^2 \left(\frac{\pi}{3}\right)$ $= 96\pi \text{ cm}^2$ (b) Circumference = arc length of sector $= 24 \left(\frac{\pi}{3}\right)$ $= 8\pi \text{ cm}$ $2\pi r = 8\pi$ $r = 4 \text{ cm}$ (c) Height of cone: $\sqrt{24^2 - 4^2} = \sqrt{576 - 16} = \sqrt{560} \text{ cm}$ Volume of cone: $\frac{1}{3}\pi(\sqrt{560})(4)^2 = \frac{1}{3}\pi(\sqrt{560})(16) \approx 396 \text{ cm}^3$	
3	(a) $h^2 + 8^2 = 17^2$ $h^2 = 17^2 - 8^2$ $h = 15 \text{ (shown)}$ (b) $\pi(8)(17) + \frac{1}{2}(4)\pi(8)^2 \approx 829$ (c) $\frac{1}{3}\pi(8)^2(15) + \frac{1}{2}\left(\frac{4}{3}\right)\pi(8)^3$ (Accept also if students assume vol of solid is V and vol of ornament would be $\frac{V}{n}$.) ≈ 2077.63994 Let height of ornament be h and n be the number of ornaments. Since solids are geometrically similar,	

	$\left(\frac{h}{23}\right)^3 = \frac{\left(\frac{2077.63994}{n}\right)}{2077.63994}$ $\left(\frac{h}{23}\right)^3 = \frac{1}{n}$ $n = \left(\frac{23}{h}\right)^3$ Since $h < 0.1 \text{ m}, n > \left(\frac{23}{0.1}\right)^3 \rightarrow n > 12167000$ We could produce 12500000 ornaments. (Accept any ans more than 12167000)	
4	 Let the radius of each quadrant be $r \text{ cm}$. The 4 arcs will form the circumference of a circle of radius $r \text{ cm}$. Perimeter of remaining shape = $(4 \times 3\pi) \text{ cm}$ $2\pi r = 12\pi$ $r = \frac{12}{2}$ $= 6 \text{ cm}$ Area of remaining shape $= \left(\begin{matrix} \text{Area of square} \\ \text{of side } 2r \end{matrix}\right) - \left(\begin{matrix} \text{Area of circle} \\ \text{of radius } r \end{matrix}\right)$ $= (2r)^2 - \pi r^2$ $= 4r^2 - \pi r^2$ $= r^2(4 - \pi)$ $= 6^2(4 - \pi) \leftarrow \text{Substitute } r = 6 \text{ cm}$ $= 36(4 - \pi) \text{ cm}^2$	
5	(a) (i) Volume of 1 ball = $\frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$ Volume of 4 balls = $144\pi \text{ cm}^3$ (ii) Width of box = $3 \times 4 = 12$ Height of box = $3 \times 2 = 6$	

	Volume of box = $12 \times 12 \times 6 = 864 \text{ cm}^3$ (iii) Volume not occupied = $864 - 144\pi = 411.611$ Percentage: $\frac{411.611}{864} \times 100\% = 47.6\%$ (b)(i) Area = $\frac{1}{3}\pi r^2$ $144\pi = \frac{1}{3}\pi(x)^2 \times 12$ $x = 6 \text{ cm}$ (shown) (ii) Slant height calculation: $l = \sqrt{12^2 + 6^2} = 13.4164$ Total surface area = $\pi r l + \pi r^2$ $= \pi(6)(13.4164) + \pi(6)^2$ $= 365.99 \text{ cm}^2$ $= 366 \text{ cm}^2$ (3 sf)	
6	(a) (i) Area of the curved surface STUV The curved surface STUV is a portion of a cylinder. The cross-section is a sector of a circle with radius 5 cm and angle 30°. First, find the arc length of the sector, which will be the width of the curved surface. The circumference of the full circle is $(2\pi r = 2\pi(5) = 10\pi) \text{ cm}$. Since the angle is 30°, which is $(\frac{30}{360} = \frac{1}{12})$ of the full circle, the arc length is $(\frac{1}{12} \times 10\pi = \frac{5\pi}{6}) \text{ cm}$. The height of the curved surface is given as 7 cm (the distance between the horizontal cross-sections OST and RVU). The area of the curved surface STUV is then the arc length multiplied by the height: $(\frac{5\pi}{6} \times 7 = \frac{35\pi}{6}) \text{ cm}^2$. (a) (ii) The angle UST is 69.7°. (b) Ratio of volumes of similar solids The two solids are geometrically similar. The ratio of their corresponding lengths (radii) is $\frac{3}{5}$. The ratio of their volumes is the cube of the ratio of their corresponding lengths. Therefore, the ratio of the volume of the smaller solid to the volume of the larger	

	solid is $\left(\frac{3}{5}\right)^3 = \frac{27}{125}$ Answer is 27:125	
7	(a) Find the total height of the cake The tiers are geometrically similar, so the ratio of all corresponding lengths is the same. - Lower tier diameter: $6y$ cm - Upper tier diameter: $4y$ cm Ratio of diameters: Ratio = $\frac{4y}{6y} = \frac{2}{3}$ Let h_1 be the height of the upper tier, and $h_2 = 12$ cm be the height of the lower tier. Since the tiers are similar, $\frac{h_1}{h_2} = \frac{\text{diameter of upper}}{\text{diameter of lower}} = \frac{2}{3}$ So, $h_1 = \frac{2}{3} \times 12 = 8 \text{ cm}$ Total height: $\text{Total height} = h_1 + h_2 = 8 + 12 = \mathbf{20 \text{ cm}}$ (b) Given that the cake weighs a total of 2 kg, find the weight of the lower tier The weights are proportional to the volumes. For similar solids: $\frac{\text{Volume of upper}}{\text{Volume of lower}} = \left(\frac{\text{linear scale}}{\text{linear scale}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$ Let W_1 be the weight of the upper tier and W_2 be the weight of the lower tier. Let total weight = $W_1 + W_2 = 2$ kg. $\frac{W_1}{W_2} = \frac{8}{27}$ $W_1 = \frac{8}{27} W_2$ $\frac{8}{27} W_2 + W_2 = 2$	

	$\left(\frac{8}{27} + 1\right) W_2 = 2$ $\left(\frac{8 + 27}{27}\right) W_2 = 2$ $\frac{35}{27} W_2 = 2$ $W_2 = 2 \times \frac{27}{35} = \frac{54}{35} \approx \mathbf{1.54kg}$	
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