

9 ESSENTIAL QUESTIONS TO MASTER

HCF & LCM

1	Determine whether each of the following is a prime or a composite number. (a) 667 (b) 677 (c) 2021 (d) 2027	
2	(a) If p and q are whole numbers such that $p \times q = 37$, find the value of $p + q$. Explain your answer. (b) If n is a whole number such that $n \times (n + 42)$ is a prime number, find the prime number. Explain your answer.	
3	Kate wishes to cut some squares from a vanguard sheet with a length of 64 cm and a breadth of 48 cm. She likes the squares to be as big as possible and she does not want any leftover vanguard sheet. (i) What is the length of each square? (ii) How many squares can she cut altogether?	
4	When written as the product of their prime factors $p = 2^3 \times 3^6$, $q = 2 \times 3^2 \times 5$, $r = 2^2 \times 3 \times 7$. Find (a) the value of the cube root of p , (b) the LCM of p , q and r , giving your answer as the product of its prime factors, (c) the greatest number that will divide p , q and r exactly.	
5	(a) Written as a product of its prime factors, $1008 = 2^x \times 3^y \times 7$. Find the values of x and y . (b) Written as a product of its prime factors, $1350 = 2 \times 3^3 \times 5^2$. Find the smallest positive integer k such that $\frac{1350}{k}$ is a square number.	
6	(a) Express 4312 as a product of its prime factors. (b) Given $588 = 2^2 \times 3 \times 7^2$. Find (i) The largest integer which is a factor of both 588 and 4312. (ii) The smallest integer which is the multiple of both 588 and 4312. (iii) The smallest integer value of m such that $4312m$ is a perfect square.	
7	Raj was trying to get to sleep one night but there was too noise around him. The noise came from his clock ticking every 20 seconds, a leaking tap in the kitchen dripping every	

	15 seconds and his father snoring every 27 seconds. He first noticed that all three events happened together at 12 midnight. (i) When will all the three events next happen together? Express your answer in minutes. (ii) How many times would all three events happen together between midnight and 1 am?	
8	Written as a product of its prime factors, $p = 2^1 \times 3^x \times 7^y$ and $q = 3 \times 7^2 \times 11$ (a) Find the smallest value of x and y for which p is a multiple of 21. (b) Explain why $33q$ is a perfect square.	
9	(a) Find the prime factors of 3150, giving your answer in index form. (b) The highest common multiple of these two numbers is 3150. Both numbers are greater than their highest common factor. Find the two numbers.	

Answer Key:

1	(a) $\sqrt{667} = 25.8$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{667}$ is 23. 667 is an odd number, so it is not divisible by 2. Since the sum of the digits of 667 is $6 + 6 + 7 = 19$ which is not divisible by 3, then 667 is not divisible by 3. The last digit of 667 is neither 0 nor 5, so 667 is not divisible by 5. A calculator may be used to test whether 667 is divisible by prime numbers more than 5. Since 667 is divisible by 23, therefore 667 is a composite number. (b) $\sqrt{677} = 26.0$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{677}$ is 23. Since 677 is not divisible by any of the prime numbers 2, 3, 5, 7, ..., 23, then 677 is a prime number. (c) $\sqrt{2021} = 45.0$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2021}$ is 43. 2021 is an odd number, so it is not divisible by 2. Since the sum of the digits of 2021 is $2 + 0 + 2 + 1 = 5$ which is not divisible by 3, then 2021 is not divisible by 3. The last digit of 2021 is neither 0 nor 5, so 2021 is not divisible by 5. A calculator may be used to test whether 2021 is divisible by prime numbers more than 5. Since 2021 is divisible by 43, therefore 2021 is a composite number. (d) $\sqrt{2027} = 45.0$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2027}$ is 43. Since 2027 is not divisible by any of the prime numbers 2, 3, 5, 7, ..., 43, then 2027 is a	
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	prime number.	
2	(a) Since 37 is a prime number, then 1 and 37 are its only two factors. It does not matter whether p or q is 1 or 37 as we only want to find the value of p+q. Therefore, $p+q = 1+37 = 38$ (b) Since $n \times (n + 42)$ is a prime number, then n and n+42 are its only two factors. Since 1 has to be one of its two factors, then $n=1$. Therefore, $n \times (n + 42) = 1 \times (1 + 42) = 1 \times 43 = 43$	
3	(a) $64 = 2^6$ $48 = 2^4 \times 3$ HCF of 64 and 48 = 2^4 $= 16$ Length of each square = 16 cm (b) Number of squares that can be cut altogether = $\frac{64}{16} \times \frac{48}{16}$ $= 4 \times 3$ $= 12$	
4	(a) $\sqrt[3]{p} = \sqrt[3]{2^3 \times 3^6}$ $= (2^3 \times 3^6)^{\frac{1}{3}}$ $= (2^3)^{\frac{1}{3}} \times (3^6)^{\frac{1}{3}}$ $= 2^1 \times 3^2$ $= 2 \times 9$ $= 18$ (b) LCM of p, q and r = $2^3 \times 3^6 \times 5 \times 7$ (c) Greatest number that will divide p, q and r exactly $=$ HCF of p, q and r $= 2 \times 3$ $= 6$	
5	(a) $1008 = 2^4 \times 3^2 \times 7$ $x = 4$ and $y = 2$ (b) Smallest positive integer $k = 2 \times 3 = 6$	
6	(a) $2^3 \times 7^2 \times 11$ (b)(i) 196 (ii) 12936	

	(iii) 22	
7	(i) $15 = 3 \times 5$ $20 = 2 \times 2 \times 5$ $27 = 3 \times 3 \times 3$ $\text{LCM of } 15, 20 \text{ and } 27 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$ The next event will happen 540 seconds or 9 minutes later, i.e. at 12.09 am. (ii) Since it happens after every 9 minutes and there are 60 minutes between midnight and 1 am, it will happen for another 6 times.	
8	(a) Smallest $x = 1$, Smallest $y = 1$ (b) Since the indices of the prime factors are multiples of 2, $33q$ is a perfect square.	
9	(a) We can find the prime factorization of 3150 by dividing by prime numbers: $3150 = 2 \times 1575$ $1575 = 3 \times 525$ $525 = 3 \times 175$ $175 = 5 \times 35$ $35 = 5 \times 7$ Therefore, $3150 = 2 \times 3^2 \times 5^2 \times 7$. (b) Let the two numbers be a and b. We are given that $\text{HCF}(a, b) = 105$ and $\text{LCM}(a, b) = 3150$. We also know that $a > 105$ and $b > 105$. We know that for any two numbers a and b, $a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b)$. So, $a \times b = 105 \times 3150 = 330750$. Since $\text{HCF}(a, b) = 105$, we can write $a = 105x$ and $b = 105y$, where x and y are coprime integers (i.e., $\text{HCF}(x, y) = 1$). Substituting these into the equation $a \times b = 330750$. $(105x)(105y) = 330750$ $11025xy = 330750$ $xy = \frac{330750}{11025} = 30$ Since x and y are coprime, we need to find pairs of coprime factors of 30. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30. Possible pairs of factors are (1, 30), (2, 15), (3, 10), (5, 6). We check which pairs are coprime: $\text{HCF}(1, 30) = 1$ $\text{HCF}(2, 15) = 1$ $\text{HCF}(3, 10) = 1$ $\text{HCF}(5, 6) = 1$ So, all the pairs are coprime. Now we find the corresponding values of a and b : If $x = 1, y = 30$, then $a = 105(1) = 105$ and $b = 105(30) = 3150$. But $a > 105$. This case is invalid. If $x = 2, y = 15$, then $a = 105(2) = 210$ and $b = 105(15) = 1575$. Both numbers are	

greater than 105. If $x = 3, y = 10$, then $a = 105(3) = 315$ and $b = 105(10) = 1050$. Both numbers are greater than 105. If $x = 5, y = 6$, then $a = 105(5) = 525$ and $b = 105(6) = 630$. Both numbers are greater than 105. Therefore, the possible pairs are (210, 1575), (315, 1050), and (525, 630).	
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