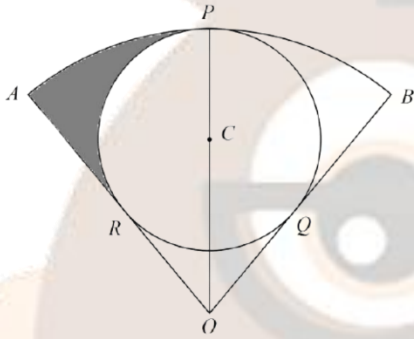
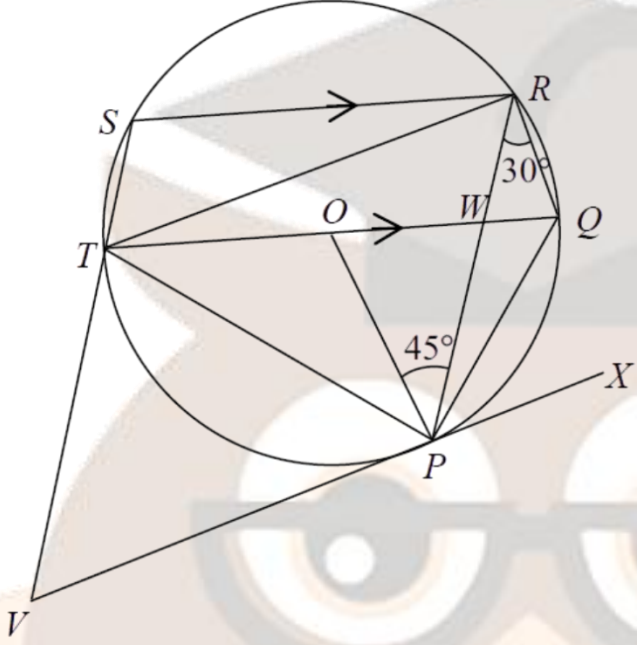
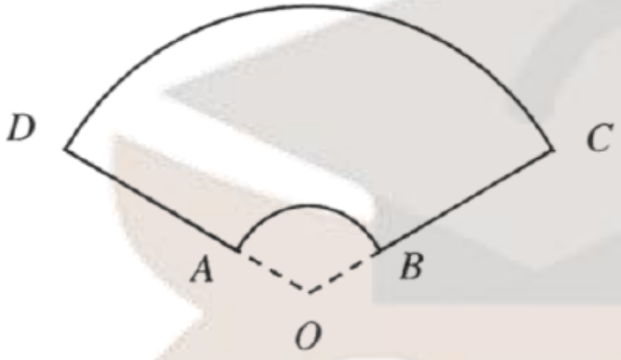
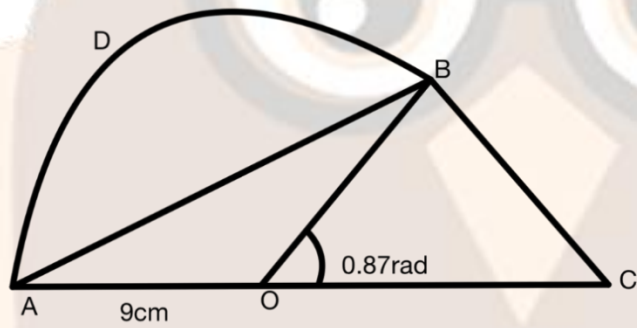


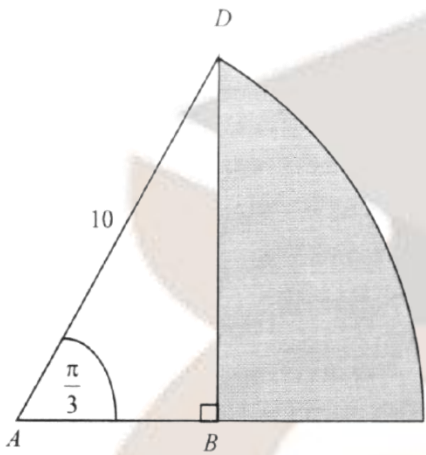
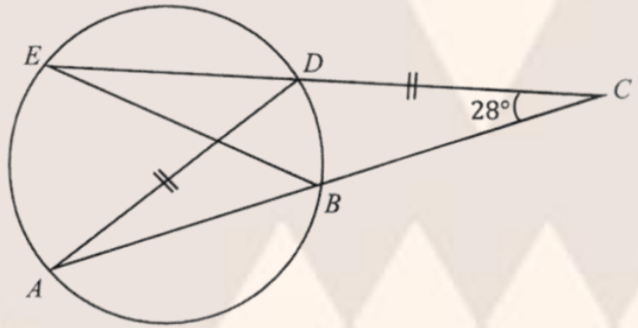
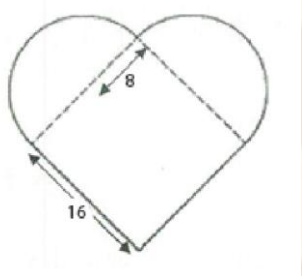
7 ESSENTIAL QUESTIONS TO MASTER

ARC SECTOR SEGMENT

1	In the diagram, $OAPB$ is a sector of a circle with center O and radius 9 cm. $\angle AOB = 1.4$ radians. C is the center of the circle enclosed inside the sector. OCP is a straight line, and the circle touches the sector at points P, Q and R  (i) Show that the radius of the enclosed circle is 3.526 cm, correct to 3 decimal places. (ii) Calculate the area of shaded region.	
2	Points P, Q, R, S, and T lie on a circle with center O. VX is tangent to the circle at P. Line SR is parallel to the diameter TQ. W lies on TQ and PR.	

	 It is given that $\angle OPR = 45^\circ$ and $\angle PRQ = 30^\circ$. (a) (i) Find $\angle TOP$. Give a reason for each step of your working. (ii) Find $\angle TPV$. Give a reason for each step of your working. (iii) Find $\angle TSR$. Give a reason for each step of your working. (b) Show that SRWT is a parallelogram.	
3	The diagram shows a mirror ABCD. AB and DC are arcs of circles center O with radii 15 cm and 45 cm respectively. The perimeter of the mirror is 286 cm.	

	 (a) Calculate the exact value of angle AOB in radians. (b) Calculate the area of the mirror.	
4	In the figure, OADB is a sector of a circle with center O and radius 9 cm. AOC is a straight line such that $\angle BOC = 0.87 \text{ rad}$.  Calculate (a) The length of the arc ADB (b) The area of segment ADB	
5	The following diagram shows a sector ACD with radius 10 cm and $\angle DAC = \frac{\pi}{3} \text{ radian}$. B is a point on AC such that DB is perpendicular to AC.	

	 (a) Convert $\frac{\pi}{3}$ radians to degrees. (b) Calculate the area of sector ACD. (c) Calculate the length AB (d) Find the perimeter of the shaded region BCD.	
6	A, B, D and E are points on the circle such that $AD = CD$ and angle $BCD = 28^\circ$. Explain with geometrical reasons, why the length of BC equals to BE. 	
7	 Instructions for baking a heart-shaped cake: • Bake a square cake of side 16 cm • Bake two semicircle cakes of radius 8 cm. • Join the two halves to the square cake, as shown in the diagram.	

	(a) Find the area of the top view of the cake.  (b) A red ribbon is fixed around the sides for the heart-shaped cake with the sides overlapping by 2 cm. Find the length of ribbon required.	
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Answer Key:

1	(i) Let the radius of the enclosed circle be r cm. $\angle CRO = 90^\circ$ (<i>tangent $\perp$ radius</i>) $\angle ROC = \frac{1.4}{2}$ (tangents from external point) $= 0.7$ rad $\sin \angle ROC = \frac{RC}{CO}$ $\sin 0.7 = \frac{r}{9 - r}$ $9 \sin 0.7 - r \sin 0.7 = r$ $r = \frac{9 \sin 0.7}{1 + \sin 0.7}$ $= 3.5262$ (4 dp) $= 3.526$ (3 dp) (ii) $\angle RCO = \pi - \frac{\pi}{2} - 0.7$ (angle sum of triangle) $\angle RCO = \frac{\pi}{2} - 0.7$ $\angle RCP = \pi - \left(\frac{\pi}{2} - 0.7\right)$ (adjacent angles on a straight line) $\angle RCP = \frac{\pi}{2} + 0.7$ Area of sector RCP $= \frac{1}{2} \left(\frac{\pi}{2} + 0.7\right) (3.526)^2 = 14.116 \text{ cm}^2$ (5 sf) Area of triangle RCO $= \frac{1}{2} (3.526)(9 - 3.526) \sin \left(\frac{\pi}{2} - 0.7\right) = 7.3812 \text{ cm}^2$ (5 sf) Area of sector OAP $= \frac{1}{2} (0.7)(9)^2 = 28.35 \text{ cm}^2$ Area of shaded region $= 28.35 - 7.3812 - 14.116 = 6.85 \text{ cm}^2$ (3 sf)	
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2	(ai) $\angle POQ = 60^\circ$ ($\angle$ at center = 2 $\angle$ at circumference) $\angle TOP = 180^\circ - 60^\circ$ (adjacent $\angle$s on a straight line) $= 120^\circ$ (a ii) $\angle OPT = \frac{180^\circ - 120^\circ}{2}$ (base $\angle$s of isosceles $\triangle$) $= 30^\circ$ $\angle OPV = 90^\circ$ (tangent $\perp$ radius) $\angle TPV = 90^\circ - 30^\circ$ $= 60^\circ$ (a iii) $\angle TSR = 180^\circ - 45^\circ - 30^\circ$ ($\angle$s in opposite segments) $= 105^\circ$ (b) $\angle TWR = 60^\circ + 45^\circ$ (exterior angle of a triangle) $\angle TWR = 105^\circ$ $\angle TWR = \angle TSR = 105^\circ$ $\angle STW = 180^\circ - 105^\circ$ (interior angles, $SR \parallel TW$) $\angle STW = 75^\circ$ $\angle SRW = 360^\circ - 75^\circ - 105^\circ - 105^\circ$ (angle sum of quadrilateral) $\angle SRW = 75^\circ$ Since opposite angles of the quadrilateral are equal ($\angle TWR = \angle TSR = 105^\circ$ and $\angle STW = \angle SRW = 75^\circ$), SRWT is a parallelogram.	
3	(a) Given Perimeter = 286 cm Arcs length = $296 - 2(30) = 226$ cm $\frac{A\hat{O}B}{2\pi} [2\pi(15) + 2\pi(45)] = 226$ $A\hat{O}B(15 + 45) = 226$ $A\hat{O}B = \frac{113}{30} = 3\frac{23}{30}$ radians (b) Area of the mirror $= \frac{A\hat{O}B}{2\pi} [\pi(45)^2 - \pi(15)^2]$ $= \frac{A\hat{O}B}{2} (45^2 - 15^2)$ $= \frac{113}{2(30)} (30)(60)$ $= 113(30)$	

	$= 3390 \text{ cm}^2$	
4	(a) $9(x - 0.87) \approx 20.4 \text{ cm}$ (b) $\frac{1}{2}(9)^2(\pi - 0.87) - \frac{1}{2}(9)(9) \sin(\pi - 0.87) \approx 61.0 \text{ cm}^2$	
5	(a) $\pi \text{ radian} = 180^\circ$ $\frac{\pi}{3} = \frac{180}{3} = 60^\circ$ (b) Area of sector $(= \frac{1}{2}r^2\theta)$ $(= \frac{1}{2} \times 10^2 \times \frac{\pi}{3})$ or $(\frac{60}{360} \times \pi(10)^2)$ $(= 52.3599 \text{ cm}^2 = 52.4 \text{ cm}^2) \text{ (3 sf)}$ (c) $\cos \frac{\pi}{3} = \frac{AB}{10}$ $AB = 5 \text{ cm}$ (d) $\sin \frac{\pi}{3} = \frac{BD}{10}$ $BD = 8.6603 \text{ cm}$ $BC = 10 - 5 = 5 \text{ cm}$ arc length $= r$ $= 10 \times \frac{\pi}{3}$ $= 10.4720 \text{ cm}$ Perimeter $= 8.6603 + 10.4720 + 5 = 24.1323$ $= 24.1 \text{ cm (3sf)}$	
6	$\angle DAC = 28^\circ$ (base angles of isosceles triangle ACD) $\angle DEB = \angle DAC$ (angles in same segment) $= 28^\circ$ $= \angle BCD$ So $\triangle BCE$ is an isosceles triangle. Therefore the length of BC equals to BE.	
7	Instructions for baking a heart-shaped cake • Bake a square cake of side 16 cm • Bake two semicircle cakes of radius 8 cm • Join the two halves to the square cake, as shown in the diagram. (a) Find the area of the top view of the cake. Step 1: Area of the square $\text{Area}_{\text{square}} = (\text{side})^2 = 16^2 = 256 \text{ cm}^2$ Step 2: Area of the two semicircles	

Each semicircle has radius $r = 8$ cm. Area of a full circle: $A_{\text{circle}} = \pi r^2 = \pi(8)^2 = 64\pi \text{ cm}^2$ Area of one semicircle: $A_{\text{semicircle}} = \frac{1}{2}\pi r^2 = 32\pi \text{ cm}^2$ Area of two semicircles: $2 \times 32\pi = 64\pi \text{ cm}^2$ Step 3: Total area Total area = $\text{Area}_{\text{square}} + \text{Area}_{\text{semicircle}} = 256 + 64\pi$ If you want a decimal answer, use $\pi \approx 3.14$: $64\pi \approx 64 \times 3.14 = 200.96$ Total area $\approx 256 + 200.96 = 456.96 \text{ cm}^2$ Final Answer for (a): $256 + 64\pi \text{ cm}^2$ or 456.96 cm^2 b) A red ribbon is fixed around the sides for the heart-shaped cake with the sides overlapping by 2 cm. Find the length of ribbon required. Step 1: Find the perimeter to be covered - Two straight sides of the square (excluding the top, since the semicircles are attached to the top side) - The perimeters of the two semicircles (which together make a full circle) Square sides: Each side = 16 cm Two sides: $2 \times 16 = 32 \text{ cm}$ Semicircle perimeters: Circumference of a full circle of radius 8 cm: $C = 2\pi r = 2\pi \times 8 = 16\pi \text{ cm}$ Total perimeter: Total length = $32 \text{ cm} + 16\pi \text{ cm}$ $16\pi \approx 16 \times 3.14 = 50.24 \text{ cm}$ Total length $\approx 32 + 50.24 = 82.24 \text{ cm}$ Step 2: Subtract the overlap Overlap = 2 cm Required ribbon length = Total length – Overlap = $82.24 - 2 = 80.24 \text{ cm}$ Final Answer for (b): $32 + 16 - 2 \text{ cm}$ or 80.24 cm	
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