

7 ESSENTIAL QUESTIONS TO MASTER

ALGEBRA

1	(a) Given that $3^{27} \div 27^3 = 3^k$, find k (b) Simplify $\frac{4x^{-4}}{y^{-\frac{4}{3}}} \times \frac{y^{\frac{2}{3}}}{18} \div \frac{1}{27}$ leaving your answers in positive indices.	
2	(a) Factorise $9a^8 + 24a^4b + 16b^2$ (b) Hence, factorise $144a^8 - (9a^8 + 24a^4b + 16b^2)$ completely	
3	(a) $\frac{5^b}{25^{a-1}} = 125^c$ Find an expression for a in terms of b and c . (b) Expand and simplify the following expression $3p^2 - (4 - 5p)(4p + 3)$	
4	(a) Factorise $35ad - 21bd + 10ac - 6bc$ completely (b) Simplify $2x < 3x + 1 \leq 13$	
5	$p = \frac{1}{2} \sqrt{\frac{x^2 - 3y}{x^2}}$ (a) Evaluate p when $x = -12$ and $y = 4$, giving your answer correct to two decimal places. (b) Rearrange the formula to make x the subject.	
6	Ms Lee goes on a vacation for d days and take \$ \$m\$ with her to spend. She spends \$ \$s\$ each day for the first w days. Write down expressions for (a) The number of dollars she has left after w days (b) The number of dollars she may spend each day for the rest of her vacation, without running out of money	
7	(a) Factorise fully $(x + y - z)^2 - (x - y + z)^2$ (b) Calculate the value of k if $x^2 + 8x + k$ is a perfect square.	

Answer Key:

1	(a) $3^{27} \div 3^9 = 3^k$	
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	$3^{27-9} = 3^k$ $k=18$ (b) $\frac{4y^{\frac{4}{3}}}{x^4} \times \frac{y^{\frac{2}{3}}}{18} \times 27$ $= \frac{6y^{\frac{6}{3}}}{x^4}$ $= \frac{6y^2}{x^4}$	
2	(a) $9a^8 + 24a^4b + 16b^4 = (3a^4 + 4b)^2$ (b) $144a^8 - (3a^4 + 4b)^2$ $= (12a^4 - (3a^4 + 4b))(12a^4 + (3a^4 + 4b))$ $= (12a^4 - 3a^4 - 4b)(12a^4 + 3a^4 + 4b)$ $= (9a^4 - 4b)(15a^4 + 4b)$	
3	(a) $\frac{5^b}{5^{2a-2}} = 5^{3c}$ $5^{b-2a+2} = 5^{3c}$ $a = \frac{b+2-3c}{2}$ OR $\frac{5^b}{5^{2a-2}} = 5^{3c}$ $5^b = 5^{3c} \times 5^{2a-2}$ $5^b = 5^{3c+2a-2}$ $b = 3c + 2a - 2$ $a = \frac{b+2-3c}{2}$ (b) $23p^2 - p - 12$	
4	(a) $(5a - 3b)(7d + 2c)$ (b) $2x < 3x + 1 \leq 13$	

	$2x < 3x + 1$ $-x < 1$ $x > -1$ $3x + 1 \leq 13$ $3x \leq 12$ $x \leq 4$ $-1 < x \leq 4$	
5	(a) $p = 0.48$ (b) $x = \pm \sqrt{\frac{3y}{1-4p^2}}$	
6	(a) She start with \$ \$m\$. She spends \$ \$s each day for the first few w days, so her total spending is sw. Therefore, the amount of money she has left is: $m - sw$ (b) the total length of her vacation is d days. She has already spent w days, so the number of days remaining is $d - w$. The amount of money she has left is $m - sw$. Therefore, the amount she can spend each day for the rest of her vacation is : $\frac{m-sw}{d-w}$	
7	(a) this is a difference of squares, so we can use the formula $A^2 - B^2 = (A + B)(A - B)$, where $A = (x + y - z)^2$ and $B = (x - y + z)$. $(x + y - z)^2 - (x - y + z)^2 = [(x + y - z) + (x - y + z)][(x + y - z) - (x - y + z)]$ $= [x + y - z + x - y + z][x + y - z - x + y - z]$ $= [2x][2y - 2z]$ $= 2x \cdot 2(y - z)$ $= 4x(y - z)$ (b) Calculate the value of k if $x^2 + 8x + k$ is a perfect square. For a quadratic expression $x^2 + bx + c$ to be a perfect square, it must be of the form $(x + a)^2 = x^2 + 2ax + a^2$. Comparing $x^2 + 8x + k$ with $x^2 + 2ax + a^2$, we can see that $b = 8$ and $c = k$. So, we have $2a = 8$, which means $a = 4$. Then, $k = a^2 = 4^2 = 16$. Therefore, the value of k is 16.	