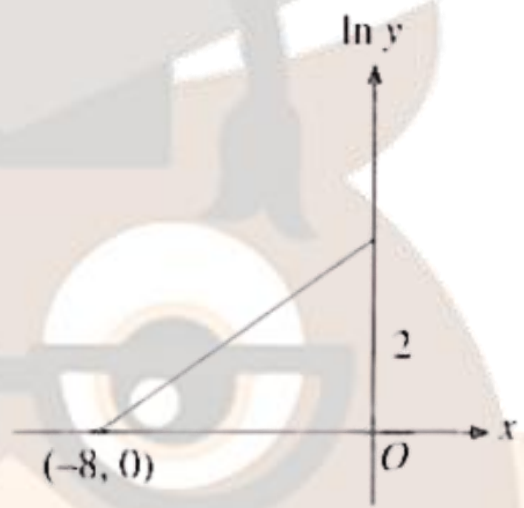


6 ESSENTIAL QUESTIONS TO MASTER

LINEAR LAW

1	Recorded values of the mass, m grams, of a radioactive substance, t hours after observations began, are shown in the table below. <table><tr><td>m (grams)</td><td>36.4</td><td>22.1</td><td>13.4</td><td>8.1</td></tr><tr><td>t (hours)</td><td>20</td><td>40</td><td>60</td><td>80</td></tr></table> It is known that m and t are related by the equation ($m = m_0e^{-kt}$), where (m_0) and k are constants. (i) Plot ($\ln m$) against t and draw a straight-line graph. Use your graph to estimate (ii) the mass of the substance when the observations began (iii) the value of k (iv) the time taken for the substance to lose half of its original mass.	m (grams)	36.4	22.1	13.4	8.1	t (hours)	20	40	60	80	
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2	The diagram shows part of a straight-line graph drawn to represent the equation ($y = \frac{h}{\sqrt{x}} + k\sqrt{x}$). Calculate the values of h and k.											
3	It is expected that the variables x and y are related by the equation ($y = \lg(ax + b)$) where a and b are unknown constants. Given sets of experimental values of x and y, determine the variables whose values should be plotted in order to obtain a straight line graph, and explain how the graph may be used to determine the values of a and b.											
4	The value, \$$V$, of a diamond is related to t, the number of years since it was mined in 1970. The variables V and t are related by the formula $V = ae^{kt}$, where a and k are constants. The table below gives the value of the diamond in 1980, 1990, 2000 and 2010. (i) On graph paper, plot $\ln V$ against t and draw a straight line graph. The vertical $\ln V$ – axis should start at 8.8 and have a scale of 2 cm to 0.1.											

	(ii) Use the graph to estimate the value of a and of k. (iii) Estimate the value of the diamond in 2014.	
5	The variables x and y are connected by the relation $y = k(ep)^x$, where k and p are constants. The graph of $\ln y$ against x is given. Find, to 3 significant figures, the values of k and p .	
6	The variables x and y are connected by an equation of the form $y = kx^n$, where k and n are constants. When $\lg y$ is plotted against $\lg x$, a straight line graph passing through $(2, 3)$ and $(8, 7)$ is obtained. Find the value of k and of n .	

Answer Key:

1	(i) Plot $(\ln m)$ against t and draw a straight-line graph. First, let's calculate $(\ln m)$ for each value of m : Let's compute the values: $\ln(36.4) \approx 3.594$ $\ln(22.1) \approx 3.095$ $\ln(13.4) \approx 2.596$ $\ln(8.1) \approx 2.092$ To plot: - x-axis: t hours - y-axis: $\ln m$ - Plot the points and draw the best-fit straight line. (ii) Estimate the mass of the substance when the observations began $t = 0$.	
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From the equation ($m = m_0 e^{-kt}$):

$$\ln m = \ln m_0 - kt$$

The y-intercept of the straight line ($t = 0$) is ($\ln m_0$).

From the table, the line passes through (20, 3.594) and (80, 2.092). Let's find the equation of the line:

Slope -k:

$$\text{slope} = \frac{2.092 - 3.594}{80 - 20} = \frac{-1.502}{60} \approx -0.0250$$

So,

$$\ln m = \ln m_0 - 0.0250t$$

Using one point to solve for ($\ln m_0$):

$$3.594 = \ln m_0 - 0.0250 \times 20$$

$$3.594 = \ln m_0 - 0.5$$

$$\ln m_0 = 3.594 + 0.5 = 4.094$$

So,

$$m_0 = e^{4.094} \approx 60.0 \text{ grams}$$

(iii) Estimate the value of k.

From above, the slope is ($-k \approx -0.0250$).

So, $k \approx 0.0250$ per hour

(iv) Estimate the time taken for the substance to lose half of its original mass (half-life).

The half-life T is when ($m = \frac{m_0}{2}$):

$$\frac{m_0}{2} = m_0 e^{-kT}$$

$$\frac{1}{2} = e^{-kT}$$

$$\ln\left(\frac{1}{2}\right) = -kT$$

$$-0.6931 = -kT$$

$$T = \frac{0.6931}{k}$$

Substitute ($k \approx 0.0250$)

$$T = \frac{0.6931}{0.0250} \approx 27.7 \text{ hours}$$

2	Let's solve for h and k using the given points and the equation: $y = \frac{h}{\sqrt{x}} + k\sqrt{x}$ Given points: (1, 8) (4, 2) Step 1: Substitute the points into the equation For (1, 8): $8 = \frac{h}{\sqrt{1}} + k\sqrt{1}$ $\rightarrow 8 = h + k$ For (4, 2): $2 = \frac{h}{\sqrt{4}} + k\sqrt{4}$ $\rightarrow 2 = \frac{h}{2} + 2k$ Step 2: Solve the system of equations From above: 1. $(h + k = 8)$ 2. $(\frac{h}{2} + 2k = 2)$ Multiply the second equation by 2 to eliminate the fraction: $h + 4k = 4$ Now subtract the first equation from this: $(h + 4k) - (h + k) = 4 - 8$ $3k = -4$ $k = -\frac{4}{3}$ Now substitute k back into the first equation: $h + \left(-\frac{4}{3}\right) = 8$ $h = 8 + \frac{4}{3} = \frac{24}{3} + \frac{4}{3} = \frac{28}{3}$
3	Given Equation $y = \lg(ax + b)$ where a and b are unknown constants. 1. Linearizing the Equation To obtain a straight-line graph, we want to express the equation in the form $y = mx + c$. Let's rewrite the equation:

	$y = \lg(ax + b)$ Recall that $\lg$ is the logarithm to base 10. 2. Choose Variables to Plot Let's let ($x = x$) and ($y = 10^y$): $y = \lg(ax + b)$ $\rightarrow 10^y = ax + b$ So, if you plot (10^y) (vertical axis) against (x) (horizontal axis), you should get a straight line. 3. Equation in Straight Line Form $10^y = ax + b$ This is of the form $y = mx + c$, where: - ($Y = 10^y$) - ($X = x$) - Slope $m = a$ - Intercept $c = b$ 4. How to Determine a and b from the Graph - The slope of the straight line gives the value of a. - The y-intercept (value of 10^y when $x = 0$) gives the value of b. Summary - Plot 10^y (vertical axis) against x (horizontal axis). - The graph will be a straight line. - The slope of the line is a. - The intercept is b.
4	(i) Plotting $\ln V$ against t First, let's calculate $\ln V$ for each value: Let's compute the values: $\begin{aligned}\ln(8000) &\approx 8.987 \\ \ln(9200) &\approx 9.128 \\ \ln(10600) &\approx 9.268 \\ \ln(12200) &\approx 9.411\end{aligned}$ To plot: - x-axis: t (10, 20, 30, 40)

- y-axis: $\ln V$ (8.987, 9.128, 9.268, 9.411)
- Draw a straight line through the points.

(ii) Estimate the value of a and k

The formula is $V = ae^{kt}$.

Taking natural logs: ($\ln V = \ln a + kt$).

This is a straight line: $y = mx + c$, where:

- ($y = \ln V$)
- ($x = t$)
- Slope ($m = k$)
- Intercept ($c = \ln a$)

Let's estimate k (slope):

$$k = \frac{\Delta(\ln V)}{\Delta t}$$

Using the first and last points:

$$k = \frac{9.411 - 8.987}{40 - 10} = \frac{0.424}{30} \approx 0.0141$$

Now, estimate $\ln a$ (intercept):

Using the point ($t = 10, \ln V = 8.987$):

$$8.987 = \ln a + (0.0141)(10)$$

$$8.987 = \ln a + 0.141$$

$$\ln a = 8.987 - 0.141 = 8.846$$

$$a = e^{8.846} \approx 6950$$

Estimated values:

- ($a \approx 6950$)
- ($k \approx 0.0141$)

(iii) Estimate the value of the diamond in 2014

2014 is ($t = 2014 - 1970 = 44$) years.

$$V = ae^{kt}$$

$$V = 6950 \times e^{0.0141 \times 44}$$

Calculate the exponent:

$$0.0141 \times 44 = 0.6204$$

$$e^{0.6204} \approx 1.85$$

$$V \approx 6950 \times 1.859 \approx 12910$$

	Estimated value in 2014: 12,900 - Plot $\ln V$ vs t and draw a straight line. $(a \approx 6950), (k \approx 0.0141)$ - Estimated value in 2014: \$12,900
5	Let's solve the problem step by step. - Given Equation $y = k(ep)^x$ - Take Natural Logarithm $\ln y = \ln[k(ep)^x] = \ln k + x \ln(ep)$ $\ln y = \ln k + x(\ln e + \ln p)$ $\ln y = \ln k + x(1 + \ln p)$ This is a straight-line equation of the form $(\ln y = mx + c)$, where: - Slope $(m = 1 + \ln p)$ - Intercept $(c = \ln k)$ - Use Points from the Graph From the graph: - When $x = 0$, $(\ln y = 2)$ - When $x = -8$, $(\ln y = 0)$ At $x = 0$: $\ln y = \ln k + 0 \times (1 + \ln p) = \ln k$ $2 = \ln k \rightarrow k = e^2$ At $(x = -8)$: $\ln y = \ln k + (-8)(1 + \ln p)$ $0 = 2 + (-8)(1 + \ln p)$ $0 = 2 - 8(1 + \ln p)$ $8(1 + \ln p) = 2$ $1 + \ln p = \frac{2}{8} = 0.25$ $\ln p = 0.25 - 1 = -0.75$ $p = e^{-0.75}$ - Final Answers (to 3 significant figures) $k = e^2 = 7.39$ (to 3 s.f.) $p = e^{-0.75} = 0.472$ (to 3 s.f.)
6	Given:

$$(y = kx^n)$$

Take logarithms (base 10, since $\lg$ is used):

$$\lg y = \lg(kx^n) = \lg k + n \lg x$$

This is in the form ($Y = c + mX$), where:

$$(Y = \lg y)$$

$$(X = \lg x)$$

$$\text{Slope } m = n$$

$$\text{Intercept } c = \lg k$$

The line passes through (2, 3) and (8, 7), where these are ($\lg x$, $\lg y$) pairs.

So:

$$\text{- When } \lg x = 2, \lg y = 3$$

$$\text{- When } \lg x = 8, \lg y = 7$$

1. Find the slope n :

$$n = \frac{7-3}{8-2} = \frac{4}{6} = \frac{2}{3}$$

2. Find the intercept $\lg k$:

Use the point (2, 3)

$$\lg y = \lg k + n \lg x$$

$$3 = \lg k + \frac{2}{3} \times 2$$

$$3 = \lg k + \frac{4}{3}$$

$$\lg k = 3 - \frac{4}{3} = \frac{9}{3} - \frac{4}{3} = \frac{5}{3}$$

3. Find k :

$$k = 10^{\frac{5}{3}}$$

Final Answers:

$$n = \frac{2}{3}, \quad k = 10^{\frac{5}{3}}$$