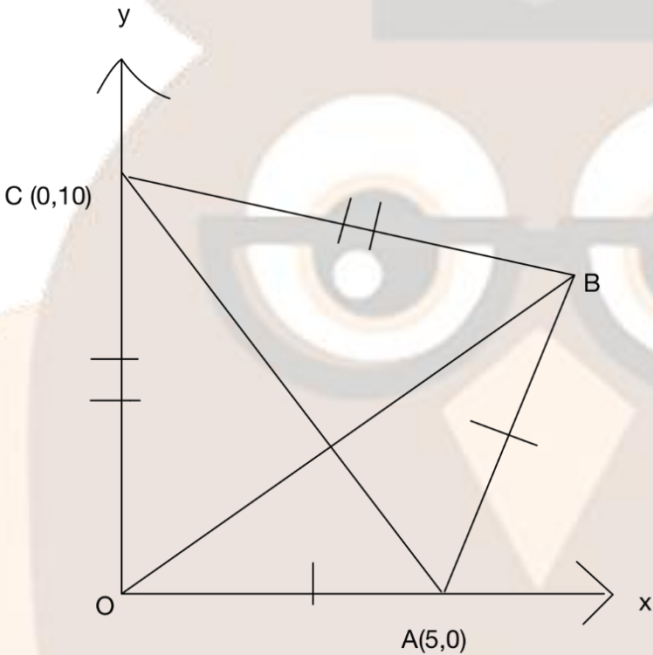
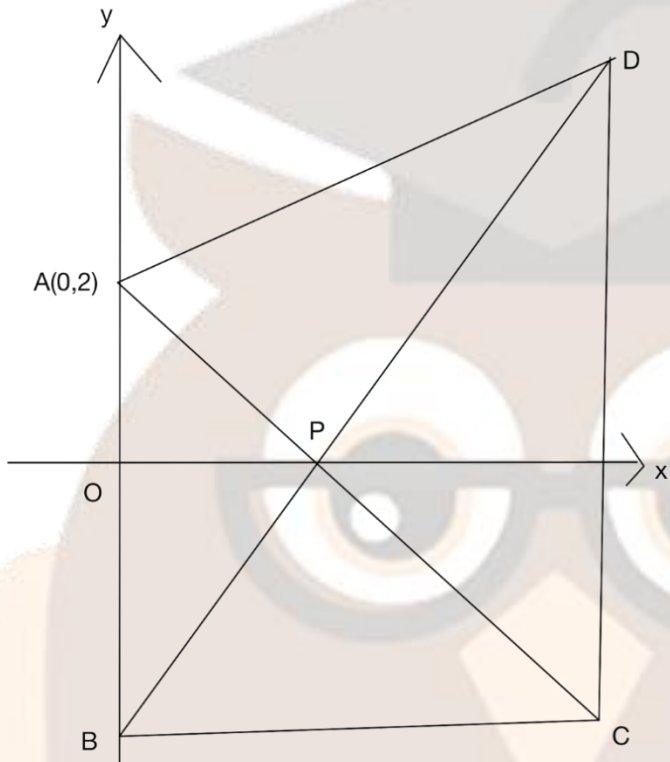
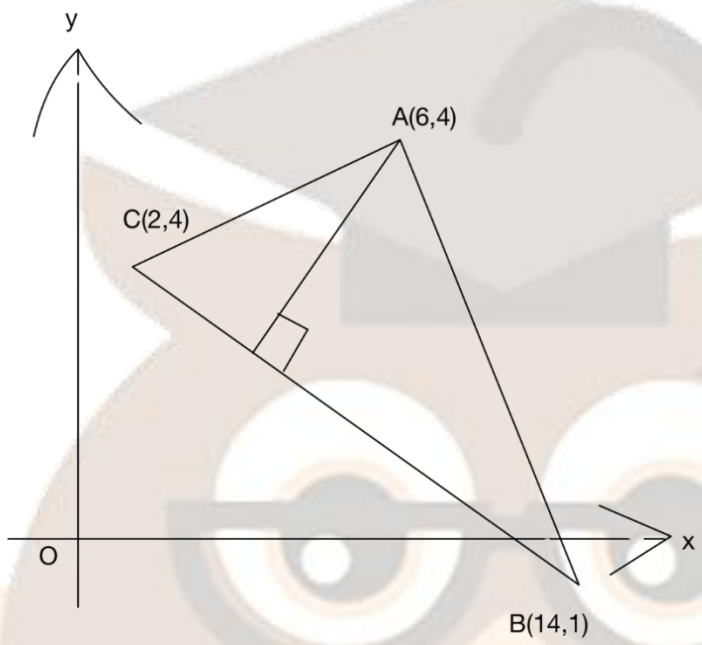
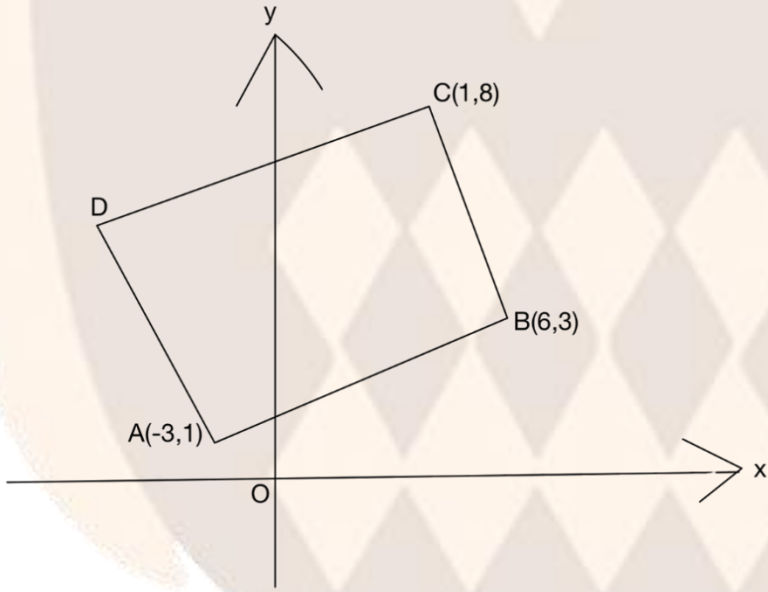


8 ESSENTIAL QUESTIONS TO MASTER

COORDINATE GEOMETRY

1	The diagram shows a kite OABC in which $OA = AB$ and $OC = BC$. Given that the coordinates of A and C are (5, 0) and (0, 10) respectively, find  (i) the equation of OB, (ii) the coordinates of B.	
2	Three points are given by A(1, 4), B(9, 8) and C(7, 12). (i) Show that angle ABC is (90°). (ii) Explain why A, B and C lie on a circle with diameter AC. (iii) Find the equation of the circle. (iv) Explain why the tangent to the circle at B is parallel to the y-axis. (v) Find the equation of the tangent to the circle at C.	
3	The diagram shows a kite ABCD which $AB = BC$ and $AD = DC$. The points A(0, 2) and B lie on the y-axis. The diagonals AC and BD intersect at the point P on the x-axis.	

	 Given that the length of AB is 4 units, (i) explain why BC is parallel to the x-axis, (ii) find the coordinates of C. Given further that the area of the kite is 28 units^2. (iii) Find the coordinates of D	
4	Calculate the area of triangle ABC shown in the diagram. Hence, find the length AF.	

		
5	Four points have coordinates A(2, -3), B(3, 0), C(0, 1), and D(-1, -2) (a) Show that ABCD is a parallelogram. (b) Calculate the length of AC and of BD. Is ABCD a rectangle?	
6	 (i) Find the gradient of BC and of AB. (ii) If H is the point on the y-axis such that B, C, and H are collinear, find the coordinates of H.	

	(iii) Find the coordinates of the point D such that ABCD is a parallelogram. (iv) Find the equation of the perpendicular bisector of BC.	
7	(a) Explain why E(k, 4) is not the midpoint of the line segment joining the points P(k, 5.5) and Q(k, 7). What can you say about the points E, P and Q? (b) Use two different methods to explain why F(-6, 1) is not the midpoint of the line segment joining the points P(-7, -1) and Q(-5, 5).	
8	The coordinates of P, Q and R are (x, y), (9, 0) and (1, 0) respectively. Given that PQ = 3PR (i) find a relationship between x and y (ii) determine whether this relationship represents a circle, showing your working clearly.	

Answer Key:

1	(i) The equation of OB: Let the origin O be at (0, 0). Since OA = AB, point B must be such that OB is a straight line and OA = AB. Also, since OC = BC, point B must be such that OC = BC. Let the coordinates of B be (x, y). Find the equation of OB: Since O = (0, 0) and B = (x, y), the equation of OB is simply the line through the origin and (x, y): $y = mx$ But we need to find the direction of OB. Notice that OA = AB and O, A, B are collinear, with A between O and B since OA = AB, so B is at twice the distance from O as A: $\overrightarrow{OB} = 2\overrightarrow{OA}$ Given A = (5, 0), so $(\overrightarrow{OA} = (5, 0))$: $\overrightarrow{OB} = 2 \times (5, 0) = (10, 0)$ So, the equation of OB is the x-axis: $y = 0$ (ii) The coordinates of B: From above, B = (10, 0). But let's check the other condition: OC = BC. O = (0, 0) C = (0, 10)	
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$B = (10, 0)$ Calculate OC: $OC = \sqrt{(0 - 0)^2 + (10 - 0)^2} = 10$ Calculate BC: $BC = \sqrt{(10 - 0)^2 + (0 - 10)^2} = \sqrt{100 + 100} = \sqrt{200} = 10\sqrt{2}$ But $(OC \neq BC)$. This suggests B is not at $(10, 0)$. Let's solve using the given conditions. Let $B = (x, y)$. 1. $OA = AB$: $OA = \sqrt{(5 - 0)^2 + (0 - 0)^2} = 5$ $AB = \sqrt{(x - 5)^2 + (y - 0)^2}$ Set $AB = OA$: $\sqrt{(x - 5)^2 + y^2} = 5$ $(x - 5)^2 + y^2 = 25$ 2. $OC = BC$: $OC = \sqrt{(0 - 0)^2 + (10 - 0)^2} = 10$ $BC = \sqrt{(x - 0)^2 + (y - 10)^2}$ Set $BC = OC$: $\sqrt{x^2 + (y - 10)^2} = 10$ $x^2 + (y - 10)^2 = 100$ Now solve equations (1) and (2): From (1): $(x - 5)^2 + y^2 = 25$ $x^2 - 10x + 25 + y^2 = 25$ $x^2 - 10x + y^2 = 0$ From (2): $x^2 + (y - 10)^2 = 100$ $x^2 + y^2 - 20y + 100 = 100$ $x^2 + y^2 - 20y = 0$ Subtract (3) from (4): $[x^2 + y^2 - 20y] - [x^2 - 10x + y^2] = 0$ $x^2 + y^2 - 20y - x^2 + 10x - y^2 =$	
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	$-20y + 10x = 10x = 20$ $x = 2y$ Substitute ($x = 2y$) into (3): $(2y)^2 - 10(2y) + y^2 =$ $4y^2 - 20y + y^2 = 0$ $5y^2 - 20y = 0$ $y(5y - 20) = 0$ So $y = 0$ or $y = 4$. If $y = 0$, $x = 0$ but B cannot be at the origin. If $y = 4$, $x = 8$. So, the coordinates of B are (8, 4). Final Answers: (i) The equation of OB: The line through O(0, 0) and B(8, 4): $\text{Slope} = \frac{4 - 0}{8 - 0} = \frac{1}{2}$ So, the equation is: $y = \frac{1}{2}x$ (ii) The coordinates of B: (8, 4)	
2	(i) Show that angle ABC is (90°). Let's find the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ $\overrightarrow{BA} = A - B = (1-9, 4-8) = (-8, -4)$ $\overrightarrow{BC} = C - B = (7-9, 12-8) = (-2, 4)$ The angle at B is (90°) if these vectors are perpendicular, i.e., their dot product is zero: $\overrightarrow{BA} \cdot \overrightarrow{BC} = (-8)(-2) + (-4)(4) = 16 - 16 = 0$ Since the dot product is zero, angle ABC is (90°). (ii) Explain why A, B and C lie on a circle with diameter AC. A triangle inscribed in a circle where one angle is (90°) must have its hypotenuse as the diameter of the circle (Thales' theorem). Since ($\angle ABC = 90^\circ$), and B is the right angle, A, B and C all lie on a circle with diameter AC.	

	(iii) Find the equation of the circle. Let's find the center and radius: - A(1, 4), C(7, 12) - The center is the midpoint of AC: Midpoint = $\left(\frac{1+7}{2}, \frac{4+12}{2}\right) = (4, 8)$ - The radius is half the distance between A and C: $AC = \sqrt{(7-1)^2 + (12-4)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ Radius = $\frac{10}{2} = 5$ So, the equation of the circle is: $(x-4)^2 + (y-8)^2 = 25$ (iv) Explain why the tangent to the circle at B is parallel to the y-axis. The tangent at B is perpendicular to the radius OB, where O is the center (4, 8). - Slope of OB: $(m = \frac{8-8}{9-4} = 0)$ - So, the radius at B is horizontal (slope 0), so the tangent is vertical (undefined slope), i.e., parallel to the y-axis. (v) Find the equation of the tangent to the circle at C. - Center O(4, 8), point C(7, 12) - Slope of OC: $(m = \frac{12-8}{7-4} = \frac{4}{3})$ - The tangent at C is perpendicular, so its slope is $(-\frac{3}{4})$. Equation of tangent at C(7, 12): $y - 12 = -\frac{3}{4}(x - 7)$ Or, rearranged: $4(y - 12) = -3(x - 7)$ $4y - 48 = -3x + 21$ $3x + 4y = 69$	
3	Given: - Kite ABCD with AB = BC and AD = DC - A(0, 2), B lies on the y-axis - Diagonals AC and BD intersect at P on the x-axis	

- AB = 4 units (i) Explain why BC is parallel to the x-axis Since A(0, 2) and B lies on the y-axis, let B(0, y). Since AB = 4, the distance between A and B is: $ y - 2 = 4 \longrightarrow y = 6 \text{ or } y = -2$ But since AB = BC and the kite is symmetric, let's choose B(0, 6) (the other case is similar). Now, AB = BC, and both are vertical segments (since both points are on the y-axis). For BC to be equal in length and the kite to be symmetric, C must be horizontally aligned with B, i.e., BC is parallel to the x-axis. Answer: BC is parallel to the x-axis because both A and B lie on the y-axis, and the kite's symmetry requires C to be horizontally aligned with B, making BC a horizontal line. (ii) Find the coordinates of C Since BC = AB = 4 and B(0, 6), and BC is parallel to the x-axis, C must be at (x, 6) such that the distance from B to C is 4 units. $\sqrt{(x - 0)^2 + (6 - 6)^2} = 4$ $x = 4 \text{ or } x = -4$ So, C(4, 6) or C(-4, 6). (iii) Find the coordinates of D given that the area of the kite is 28 units². Step 1: Assign coordinates From the diagram and previous work: A(0, 2) B(0, -2) since AB = 4 and both are on the y-axis C(x, -2) since BC = 4 and BC is parallel to the x-axis D(x, y) - pending Step 2: Find C Since BC = 4 and both B and C have (y = -2): $BC = x - 0 = 4 \longrightarrow x = 4 \text{ or } x = -4$ From the diagram, C is to the right, so C(4, -2).	
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Step 3: Use kite properties For a kite: - $AB = BC = 4$ - $AD = DC$ Let $D(4, y)$. Distance AD: $AD = \sqrt{(4 - 0)^2 + (y - 2)^2} = \sqrt{16 + (y - 2)^2}$ Distance DC: $DC = y + 2$ Set $AD = DC$: $\sqrt{16 + (y - 2)^2} = y + 2 $ Square both sides: $16 + (y - 2)^2 = (y + 2)^2$ $16 + y^2 - 4y + 4 = y^2 + 4y + 4$ $20 - 4y = 4y + 4$ $20 - 4 = 8y$ $16 = 8y$ $y = 2$ But this gives $D(4, 2)$, which does not satisfy the area condition. Let's check the area formula. Step 4: Area of the kite Area $(= \frac{1}{2} \times d_1 \times d_2 = 28)$ - (d_1) is the length of AC - (d_2) is the length of BD Find AC: $A(0, 2), C(4, -2)$ $AC = \sqrt{(4 - 0)^2 + (-2 - 2)^2} = \sqrt{16 + 16} = 4\sqrt{2}$ Find BD: $B(0, -2), D(4, y)$ $BD = \sqrt{(4 - 0)^2 + (y + 2)^2} = \sqrt{16 + (y + 2)^2}$	
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	Set area: $\frac{1}{2} \times 4\sqrt{2} \times \sqrt{16 + (y + 2)^2} = 28$ $2\sqrt{2} \times \sqrt{16 + (y + 2)^2} = 28$ $\sqrt{2} \times \sqrt{16 + (y + 2)^2} = 14$ $\sqrt{2(16 + (y + 2)^2)} = 14$ $2(16 + (y + 2)^2) = 196$ $16 + (y + 2)^2 = 98$ $(y + 2)^2 = 82$ $y + 2 = \pm\sqrt{82}$ $y = -2 \pm \sqrt{82}$ Step 5: Final coordinates of D So, the coordinates of D are: $(4, -2 + \sqrt{82})$ or $(4, -2 - \sqrt{82})$ But from the diagram, D is above the x-axis, so the answer is: $(4, -2 + \sqrt{82})$ Final Answer: $D(4, -2 + \sqrt{82})$	
4	1. Area of Triangle ABC Given points: -A (6, 4) -B (14, -1) -C (2, 4) The area of a triangle with vertices $((x_1, y_1)), ((x_2, y_2)), ((x_3, y_3))$ is: $\text{Area} = \frac{1}{2} x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) $ Plug in the coordinates: $\text{Area} = \frac{1}{2} 6(-1 - 4) + 14(4 - 4) + 2(4 - (-1)) $ $= \frac{1}{2} 6(-5) + 14(0) + 2(5) $	

$$= \frac{1}{2} |-30 + 0 + 10|$$

$$= \frac{1}{2} |-20| = \frac{1}{2} \times 20 = 10$$

Area of triangle ($ABC = 10$) square units.

2. Find the length AF

F is the foot of the perpendicular from A to BC.

Let's find the equation of line BC:

- Slope of BC: ($m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{14 - 2} = \frac{-5}{12}$)

- Equation of BC: ($y - 4 = m(x - 2)$)

$$y - 4 = -\frac{5}{12}(x - 2)$$

The perpendicular from A to BC will have slope ($m' = \frac{12}{5}$) (negative reciprocal).

Equation of AF through A(6, 4):

$$y - 4 = \frac{12}{5}(x - 6)$$

Set the two equations equal to find F:

$$-\frac{5}{12}(x - 2) + 4 = \frac{12}{5}(x - 6) + 4$$

Let's solve for x:

First, subtract 4 from both sides:

$$-\frac{5}{12}(x - 2) = \frac{12}{5}(x - 6)$$

Multiply both sides by 60 (LCM of 12 and 5) to clear denominators:

$$-5 \times 5(x - 2) = 12 \times 12(x - 6)$$

$$-25(x - 2) = 144(x - 6)$$

$$-25x + 50 = 144x - 864$$

$$50 + 864 = 144x + 25x$$

$$914 = 169x$$

$$x = \frac{914}{169}$$

Now, substitute x back into the equation of BC to find y:

$$y - 4 = -\frac{5}{12} \left(\frac{914}{169} - 2 \right)$$

	$= -\frac{5}{12} \left(\frac{914 - 338}{169} \right)$ $= -\frac{5}{12} \left(\frac{576}{169} \right)$ $= -\frac{5 \times 576}{12 \times 169}$ $= -\frac{2880}{2028}$ $= -\frac{160}{113}$ $y = 4 - \frac{160}{113} = \frac{452 - 160}{113} = \frac{292}{113}$ $\left(F \left(\frac{914}{169}, \frac{292}{113} \right) \right).$ 3. Length AF Distance between A(6, 4) and $F \left(\frac{914}{169}, \frac{292}{113} \right)$: $AF = \sqrt{\left(6 - \frac{914}{169} \right)^2 + \left(4 - \frac{292}{113} \right)^2}$ Let's compute these values: $-\left(6 - \frac{914}{169} \right), \text{ so } \left(6 - \frac{914}{169} \right) = \frac{100}{169}$ $-\left(4 - \frac{292}{113} \right), \text{ so } \left(4 - \frac{292}{113} \right) = \frac{160}{113}$ $AF = \sqrt{\left(\frac{100}{169} \right)^2 + \left(\frac{160}{113} \right)^2}$ Therefore, the length AF is approximately 1.53 units.	
5	(a) Show that ABCD is a parallelogram A quadrilateral is a parallelogram if both pairs of opposite sides are equal and parallel (i.e., have the same vector). Let's find the vectors: - Vector AB = (3-2, 0-(-3)) = (1, 3) - Vector CD = (-1-0, -2-1) = (-1, -3) - Vector BC = (0-3, 1-0) = (-3, 1) - Vector DA = (2-(-1), -3-(-2)) = (3, -1) Notice: Vector AB = (1, 3) and Vector CD = (-1, -3) = $-1 \times (1, 3)$ (same magnitude, opposite direction) Vector BC = (-3, 1) and Vector DA = (3, -1) = $-1 \times (-3, 1)$ (same magnitude, opposite direction)	

	So, both pairs of opposite sides are equal in length and parallel, confirming ABCD is a parallelogram. (b) Calculate the length of AC and BD Length of AC: $AC = \sqrt{(0 - 2)^2 + (1 - (-3))^2} = \sqrt{(-2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$ Length of BD: $BD = \sqrt{(-1 - 3)^2 + (-2 - 0)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$ Is ABCD a rectangle? A parallelogram is a rectangle if its diagonals are equal in length and the adjacent sides are perpendicular. From above, $AC = BD = 2\sqrt{5}$, so the diagonals are equal. Now, check if adjacent sides are perpendicular: - Vector AB = (1, 3) - Vector BC = (-3, 1) Dot product: $(1, 3) \cdot (-3, 1) = (1)(-3) + (3)(1) = -3 + 3 = 0$ Since the dot product is zero, the sides are perpendicular. Conclusion: Yes, ABCD is a rectangle.	
6	(i) Find the gradient of BC and of AB. The gradient (slope) between two points (x_1, y_1) and (x_2, y_2) is: $m = \frac{y_2 - y_1}{x_2 - x_1}$ Gradient of BC: B(6, 3), C(1, 8) $m_{BC} = \frac{8 - 3}{1 - 6} = \frac{5}{-5} = -1$ Gradient of AB: A(-3, 1), B(6, 3)	

$m_{AB} = \frac{3-1}{6-(-3)} = \frac{2}{9}$ (ii) If H is the point on the y-axis such that B, C and H are collinear, find the coordinates of H. Let H = (0, h) (since it is on the y-axis). Points B, C, and H are collinear if the gradient from B to C equals the gradient from C to H: Gradient BC = Gradient CH $\frac{8-3}{1-6} = \frac{h-8}{0-1}$ $-1 = \frac{h-8}{-1}$ $-1 = -(h-8)$ $-1 = -h+8$ $h = 9$ So, H = (0, 9). (iii) Find the coordinates of the point D such that ABCD is a parallelogram. For ABCD to be a parallelogram, $(\overrightarrow{AB} = \overrightarrow{DC})$. $\overrightarrow{AB} = (6 - (-3), 3 - 1) = (9, 2)$ Let D = (x, y). Then $(\overrightarrow{DC} = (1 - x, 8 - y))$. Set $(\overrightarrow{DC} = \overrightarrow{AB})$: $(1 - x, 8 - y) = (9, 2)$ $1 - x = 9 \rightarrow x = 1 - 9 = -8$ $8 - y = 2 \rightarrow y = 8 - 2 = 6$ So, D = (-8, 6). (iv) Find the equation of the perpendicular bisector of BC. First, find the midpoint M of BC: $M = \left(\frac{6+1}{2}, \frac{3+8}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right)$ The gradient of BC is -1, so the perpendicular gradient is the negative reciprocal: 1. Equation of the line with gradient 1 through M: $y - \frac{11}{2} = 1 \left(x - \frac{7}{2}\right)$ $y = x - \frac{7}{2} + \frac{11}{2}$ $y = x + 2$	
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7	(a) Step 1: Find the midpoint of P and Q: The midpoint M of two points (x_1, y_1) and (x_2, y_2) is: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ For P(k, 5.5) and Q(k, 7): $M = \left(\frac{k+k}{2}, \frac{5.5+7}{2} \right) = (k, 6.25)$ Step 2: Compare with E(k, 4) : - The midpoint is (k, 6.25). - E is at (k, 4). Since $(4 \neq 6.25)$, E is not the midpoint. Step 3: What can you say about the points? - All three points have the same x -coordinate k , so they are vertically aligned. - E is below both P and Q on the y-axis. - E does not lie halfway between P and Q in terms of y-coordinate. (b) Use two different methods to explain why F(-6, 1) is not the midpoint of the line segment joining the points P(-7, -1) and Q(-5, 5). Method 1: Calculate the actual midpoint $M = \left(\frac{-7+(-5)}{2}, \frac{-1+5}{2} \right) = \left(\frac{-12}{2}, \frac{4}{2} \right) = (-6, 2)$ - The actual midpoint is (-6, 2). - F is at (-6, 1). - The x -coordinates are the same, but the y-coordinates are different $(1 \neq 2)$, so F is not the midpoint. Method 2: Check distances from F to P and Q Distance from F to P: $d_1 = \sqrt{(-6 - (-7))^2 + (1 - (-1))^2} = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$ Distance from F to Q: $d_2 = \sqrt{(-6 - (-5))^2 + (1 - 5)^2} = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17}$ - The distances are not equal $(\sqrt{5} \neq \sqrt{17})$, so F is not equidistant from P and Q, and thus not the midpoint.
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8	(i) Find a relationship between x and y. Let: $(P = (x, y))$ $(Q = (9, 0))$ $(R = (1, 0))$ Given: $PQ = 3PR$ First, calculate the distances: $PQ = \sqrt{(x - 9)^2 + (y - 0)^2} = \sqrt{(x - 9)^2 + y^2}$ $PR = \sqrt{(x - 1)^2 + (y - 0)^2} = \sqrt{(x - 1)^2 + y^2}$ Given $PQ = 3PR$: $\sqrt{(x - 9)^2 + y^2} = 3\sqrt{(x - 1)^2 + y^2}$ Square both sides to eliminate the square roots: $(x - 9)^2 + y^2 = 9[(x - 1)^2 + y^2]$ Expand the right side: $(x - 9)^2 + y^2 = 9(x^2 - 2x + 1 + y^2)$ $(x - 9)^2 + y^2 = 9x^2 - 18x + 9 + 9y^2$ Expand $(x - 9)^2$: $x^2 - 18x + 81 + y^2 = 9x^2 - 18x + 9 + 9y^2$ Bring all terms to one side: $x^2 - 18x + 81 + y^2 - 9x^2 + 18x - 9 - 9y^2 = 0$ Combine like terms: $(x^2 - 9x^2) + (-18x + 18x) + (81 - 9) + (y^2 - 9y^2) = 0$ $-8x^2 + 0x + 72 - 8y^2 = 0$ So, $-8x^2 - 8y^2 + 72 = 0$ Divide both sides by -8: $x^2 + y^2 = 9$ (ii) Does this relationship represent a circle? Yes! The equation $(x^2 + y^2 = 9)$ is the equation of a circle centered at the origin $(0, 0)$ with radius 3.
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