

7 ESSENTIAL QUESTIONS TO MASTER

PROPORTION

1	The surface area of a sphere, $A \text{ cm}^2$, is directly proportional to the square of its radius, r. When $r = 4$, $A = 64\pi \text{ cm}^2$. (a) Find the equation connecting A and r. Give the value of k, the constant, in terms of π. (b) Calculate the value of A when $r = 6$, giving your answer in terms of π.	
2	The intensity of light, (I), from a point source is inversely proportional to the square of the distance, (d), from the source. At a distance of 3 meters, the intensity is 400 lux. (a) Find an equation connecting (I) and (d). (b) Find the distance from the source if the intensity is 100 lux.	
3	(a) p is inversely proportional to q^3. When q is decreased by 50%, find the percentage change in p. (b) y is proportional to the square root of x. If the value of x is increased by 300%, the value of y will be increased by $r\%$. Find the value r.	
4	(a) z is inversely proportional to the cube of w. Given that $z = 16$ for a particular value of w, find the value of z when the value of w is tripled. (b) m is directly proportional to n and inversely proportional to the square of r. Given that $m = 10$ when $n = 5$ and $r = 2$, find the value of m when n is doubled and r is halved.	
5	(a) The force, F Newtons, between any two particles in the universe is inversely proportional to the square of the distance, r metres, between them. It is known that $F = 100$ Newtons for a particular value r. Find the value of F when this value of r is doubled. (b) y is proportional to x^3. Explain the effect on the value of x in order for y to be 2700% of its original value. (c)(i) y is directly proportional to the cube root of $(x + 1)$. It is given that $x = 7$ when $y = 1$. Find the value of y when $x = 124$. (ii) F is inversely proportional to the square of d. Explain what happens to F when d is halved.	
6	The force of attraction, F (in N), between two magnets is inversely proportional to the square of the distance, x (in cm), between the magnets. When the magnets are 3 cm apart, the force is 8 N. (a) Find the equation connecting F and x. (b) Find the percentage decrease in the force of attraction when the distance between them is doubled.	

7	(a) It takes 4 workers 12 weeks to complete a landscaping project. How many extra workers are needed if the project is to be completed in 8 weeks? (b) is inversely proportional to x^2. It is given that $y = 36$ for a particular value of x. Find the value of y when x is tripled.	
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Answer Key:

1	(a) Find the equation connecting A and r: Since the surface area A is directly proportional to the square of the radius r, we can write the relationship as: $A = kr^2$ where k is the constant of proportionality. We are given that when $r = 4$, $A = 64\pi$. We can use this information to find the value of k: $64\pi = k(4^2)$ $64\pi = 16k$ $k = \frac{64\pi}{16}$ $k = 4\pi$ So, the equation connecting A and r is: $A = 4\pi r^2$ (b) Calculate the value of A when $r = 6$: Now we can use the equation we found to calculate A when $r = 6$: $A = 4\pi(6^2)$ $A = 4\pi(36)$ $A = 144\pi$ Therefore, the value of A when $r = 6$ is $144\pi \text{ cm}^2$.	
2	(a) Set up the basic equation: Since the intensity (I) is inversely proportional to the square of the distance (d), we can write: $I = \frac{k}{d^2}$ where (k) is the constant of proportionality. 2. Find the constant of proportionality, (k):	

	We are given that when ($d = 3$) meters, ($I = 400$) lux. Substitute these values into the equation: $400 = \frac{k}{3^2}$ $400 = \frac{k}{9}$ $k = 400 \times 9 = 3600$ 3. Write the equation connecting (I) and (d): Now that we have the value of (k), we can write the equation as: $I = \frac{3600}{d^2}$ This is the equation that connects the intensity (I) and the distance (d). (b) Find the distance when ($I = 100$) lux: We are asked to find the distance (d) when the intensity ($I = 100$) lux. Substitute this value into the equation: $100 = \frac{3600}{d^2}$ Now, solve for (d^2): $d^2 = \frac{3600}{100}$ $d^2 = 36$ Take the square root of both sides to find (d): $d = \sqrt{36} = 6 \text{ (Distance from the source is 6m)}$	
3	(a) 1. Express the inverse proportionality: Since p is inversely proportional to q^3, we can write this relationship as: $p = \frac{k}{q^3}$ where k is the constant of proportionality. 2. Consider the change in q: If q is decreased by 50%, the new value of q (let's call it q') is: $q' = q - 0.5q = 0.5q = \frac{1}{2}q$ 3. Determine the new value of p: Now, we need to find the new value of p, which we'll call p', when q is replaced by q':	

	$p' = \frac{k}{(q')^3} = \frac{k}{\left(\frac{1}{2}q\right)^3} = \frac{k}{\frac{1}{8}q^3} = 8 \frac{k}{q^3}$ Since $p = \frac{k}{q^3}$, we can substitute p into the equation: $p' = 8p$ 4. Calculate the percentage change in p: The change in p is $p' - p = 8p - p = 7p$. The percentage change in p is: $\frac{\text{change in } p}{\text{original } p} \times 100\% = \frac{7p}{p} \times 100\% = 700\%$ (b) $y = k\sqrt{x}$ When x is increased by 300%, it becomes 4x. $y = k\sqrt{4x}$ $y = 2(k\sqrt{x})$ y is 2 times, i.e., increased by 1 time, or 100%. $r = 100\%$	
4	(a) Since z is inversely proportional to the cube of w, we can write the relationship as: $z = \frac{k}{w^3}$ where k is the constant of proportionality. Given $z = 16$ for a particular value of w, let's call that w_1. Then: $16 = \frac{k}{w_1^3}$ So, $k = 16w_1^3$. Now, we want to find the value of z when w is tripled, i.e., $w = 3w_1$. Let's call this new value of z, z_2. $z_2 = \frac{k}{(3w_1)^3} = \frac{16w_1^3}{27w_1^3} = \frac{16}{27}$ So, the value of z when w is tripled is $\frac{16}{27}$. (b) Since m is directly proportional to n and inversely proportional to the square of r, we can write the relationship as: $m = \frac{kn}{r^2}$ where k is the constant of proportionality. Given $m = 10$ when $n = 5$ and $r = 2$, we can find k: $10 = \frac{k \times 5}{2^2} = \frac{5k}{4}$ So, $k = \frac{10 \times 4}{5} = 8$.	

	Now, we want to find the value of m when n is doubled (i.e., $n = 2 \times 5 = 10$) and r is halved (i.e., $r = \frac{2}{2} = 1$). Let's call this new value of m, m_2. $m_2 = \frac{8 \times 10}{1^2} = \frac{80}{1} = 80$ So, the value of m when n is doubled and r is halved is 80.	
5	(a) $F \propto \frac{1}{r^2}$ $F = \frac{k}{r^2}$ $100 = \frac{k}{r^2}$ $k = 100r^2$ $F_2 = \frac{k}{(r_2)^2} \quad [r_2 = 2r]$ $F_2 = \frac{100r^2}{(2r)^2}$ $F_2 = \frac{100r^2}{4r^2}$ $F_2 = 25$ (b) $y_2 = 2700\% \text{ of } y = y_2 = 27y$ $x_2 = \sqrt[3]{2700\% \text{ of } y} = x_2 = \sqrt[3]{27x}$ $x_2 = \sqrt[3]{27x}$ $x_2 = 3x$ $x^2 = 3x \text{ OR } x \text{ is tripled of its original value OR } x \text{ is } 300\% \text{ of its original value.}$ (c)(i) $y = k\sqrt[3]{x+1}$ $1 = k\sqrt[3]{7+1}$ $k = 0.5$ $y = 0.5\sqrt[3]{124+1} = 2.5$ (ii) $F = \frac{k}{d^2}$ $\text{New } F = \frac{k}{(0.5d)^2}$ $= 4\left(\frac{k}{d^2}\right) = 4F$	

	New value of y becomes 4 times of the original value. Other good answers: F increases to 400% of the original value F increases by 300% F increases by 3 times F is increased by a factor of 4 Acceptable answers: F is multiplied by 4	
6	(a) Find the equation connecting F and x: Since F is inversely proportional to the square of the distance, we can write the relationship as: $F = \frac{k}{x^2}$ where k is the constant of proportionality. We are given that when $x = 3$ cm, $F = 8$ N. Plug these values into the equation to find: $8 = \frac{k}{3^2}$ $8 = \frac{k}{9}$ $k = 8 \times 9 = 72$ So, the equation connecting F and x is: $F = \frac{72}{x^2}$ (b) Find the percentage decrease in the force of attraction when the distance between them is doubled: Let F_1 be the force when the distance is x, and F_2 be the force when the distance is doubled, i.e., $2x$. $F_1 = \frac{72}{x^2}$ $F_2 = \frac{72}{(2x)^2} = \frac{72}{4x^2} = \frac{1}{4} \cdot \frac{72}{x^2} = \frac{1}{4} F_1$ The decrease in force is $F_1 - F_2 = F_1 - \frac{1}{4} F_1 = \frac{3}{4} F_1$. The percentage decrease is: $\frac{\text{Decrease in force}}{\text{Original force}} \times 100\% = \frac{\frac{3}{4} F_1}{F_1} \times 100\% = \frac{3}{4} \times 100\% = 75\%$ Therefore, the percentage decrease in the force of attraction when the distance is doubled is 75%.	
7	(a) It takes 4 workers 12 weeks to complete a landscaping project. How many extra workers are needed if the project is to be completed in 8 weeks? Let w be the number of workers and t be the time in weeks. The amount of work done is constant, so the number of workers and the time taken are inversely proportional. Thus, $w \times t = k$, where k is a constant.	

Initially, $w = 4$ and $t = 12$, so $k = 4 \times 12 = 48$. Now, we want to find the number of workers needed to complete the project in 8 weeks. Let w' be the new number of workers and $t' = 8$ weeks. Then, $w' \times t' = k$, so $w' \times 8 = 48$. $w' = \frac{48}{8} = 6$ So, 6 workers are needed to complete the project in 8 weeks. The number of extra workers needed is $6 - 4 = 2$. (b) y is inversely proportional to x^2. It is given that $y = 36$ for a particular value of x. Find the value of y when x is tripled. Since y is inversely proportional to x^2, we have $y = \frac{k}{x^2}$, where k is a constant. We are given that $y = 36$ for a particular value of x. So, $36 = \frac{k}{x^2}$, which means $k = 36x^2$. Now, we want to find the value of y when x is tripled, i.e., when x becomes $3x$. Let y' be the new value of y. Then, $y' = \frac{k}{(3x)^2} = \frac{36x^2}{9x^2} = \frac{36}{9} = 4$ So, the value of y when x is tripled is 4.	
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