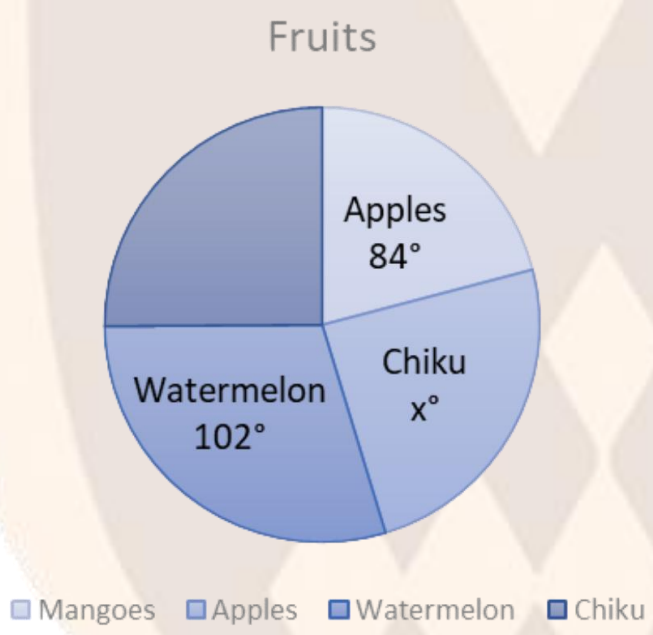


9 ESSENTIAL QUESTIONS TO MASTER

PROBABILITY & STATISTICS

1	A box contains 30 pens, m of which are black, $(m + 2)$ are purple, and the rest are green. A pen is chosen at random from the box. (a) Write down, in terms of m, the probability that the pen is green. (b) If the probability of choosing a green pen is $\frac{1}{3}$, find the number of black pens.														
2	There are $(2n + 3)$ red marbles, $(4n - 1)$ green marbles and $(18 - 2n)$ blue marbles in a bag. (a) Given that the probability of drawing a red or green marble is $\frac{19}{22}$, show that $n = 6$ (b) Hence find the probability of drawing 2 red marbles, one after another without replacement.														
3	The stem-and-leaf diagram represents the number of chocolates in some packets. <table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>2</td><td>0 2 4 4 4 5 6 6 6 6 6 7 8 9 9 9</td></tr><tr><td>3</td><td>1 1 1 2 2 3 4 4 5 5 7 7 8 8</td></tr><tr><td>4</td><td>0 1 2 3 3 3 5 5 6 8</td></tr></table> Key: 2 2 means 22 (a) Find (i) the mode, (ii) the median, and (b) Find the percentage of packets which have more than or equal to 25 chocolates but fewer than 32 chocolates.	Stem	Leaf	2	0 2 4 4 4 5 6 6 6 6 6 7 8 9 9 9	3	1 1 1 2 2 3 4 4 5 5 7 7 8 8	4	0 1 2 3 3 3 5 5 6 8						
Stem	Leaf														
2	0 2 4 4 4 5 6 6 6 6 6 7 8 9 9 9														
3	1 1 1 2 2 3 4 4 5 5 7 7 8 8														
4	0 1 2 3 3 3 5 5 6 8														
4	1000 students took a Mathematics examination which consisted of Paper 1 and Paper 2. Each paper has a total of 50 marks. The results of these students are tabulated as shown. <table><tr><th rowspan="2">Marks (x)</th><th colspan="2">Number of Candidates</th></tr><tr><th>Paper 1</th><th>Paper 2</th></tr><tr><td>$0 < x \leq 10$</td><td>90</td><td>100</td></tr><tr><td>$10 < x \leq 20$</td><td>310</td><td>270</td></tr><tr><td>$20 < x \leq 30$</td><td>370</td><td>295</td></tr></table>	Marks (x)	Number of Candidates		Paper 1	Paper 2	$0 < x \leq 10$	90	100	$10 < x \leq 20$	310	270	$20 < x \leq 30$	370	295
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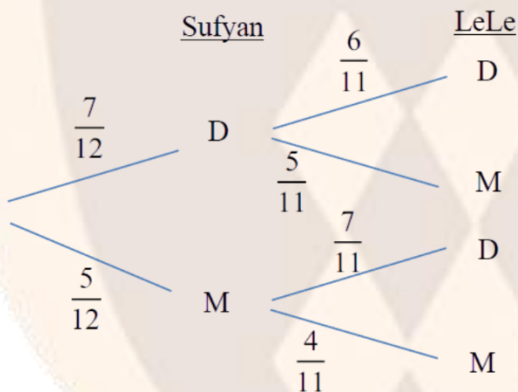
	<table border="1"> <tr> <td>$30 < x \leq 40$</td><td>200</td><td>295</td></tr> <tr> <td>$40 < x \leq 50$</td><td>30</td><td>40</td></tr> </table>	$30 < x \leq 40$	200	295	$40 < x \leq 50$	30	40	
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	(a) Calculate the mean of Paper 1 and 2. (b) Which paper is more difficult? Explain your answer.							
5	The chart shows the total number of passengers at Changi Airport in the years 2016 to February 2020.  State one feature of the chart that may be misleading and explain why.							
6	300 children were surveyed on their favourite tropical fruits. The pie chart shows the results of the survey.  (a) 60 children like rambutans, find the value of (x) (b) Find the ratio of the number of children who like rambutans to those who like mangoes. (c) Express the number of children who like durians as a percentage of the total number of							

	students surveyed.															
7	A box of chocolate contains 7 dark chocolates and 5 milk chocolates. Sufyan takes a chocolate, selected at random, from the box and eats it. LeLe then takes a chocolate, selected at random, from the box. (i) Draw a tree diagram to show the probabilities of the possible outcomes. (ii) Find, as a fraction in its simplest form, the probability that (a) Sufyan and LeLe both picked dark chocolates (b) LeLe picked a milk chocolate (c) at least one milk chocolate was chosen.															
8	The table shows information about a group of students in a class. <table><tr><td></td><td>Boys</td><td>Girls</td></tr><tr><td>Wear spectacles</td><td>4</td><td>6</td></tr><tr><td>Do not wear spectacles</td><td>16</td><td>14</td></tr></table> (i) A member of the group is selected at random. Find the probability that the student (a) is a boy, (b) wears spectacles. (ii) Calculate the number of students to be removed so that the probability of drawing a girl who wears spectacles from the remaining students in the class is $\frac{1}{6}$. (i) (a) Find the probability that the student is a boy. (b) Find the probability that the student wears spectacles (ii) Calculate the number of students to be removed so that the probability of drawing a girl who wears spectacles from the remaining students in the class is $\frac{1}{6}$.		Boys	Girls	Wear spectacles	4	6	Do not wear spectacles	16	14						
	Boys	Girls														
Wear spectacles	4	6														
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9	A six-faced fair die was thrown 20 times. The table shows the number of times that each possible score occurred. <table><tr><td>Score</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Frequency</td><td>1</td><td>4</td><td>3</td><td>2</td><td></td><td></td></tr></table> (a) Show that $x + y = 10$. (b) if the mean score is 3.7, show that $2x + 5y = 37$ (c) using a) and b), find the values of x and y. Explain why x and y cannot be the values you find.	Score	1	2	3	4	5	6	Frequency	1	4	3	2			
Score	1	2	3	4	5	6										
Frequency	1	4	3	2												

Answer Key:

1	(a) First, we need to find the number of green pens. The total number of pens is 22. The number of red pens is n, and the number of blue pens is $(n - 1)$. Therefore, the number of green pens is:	
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	Number of green pens = Total pens - (Number of red pens + Number of blue pens) $= 22 - (n + (n - 1))$ $= 22 - (2n - 1)$ $= 22 - 2n + 1$ $= 23 - 2n$ The probability of choosing a green pen is the number of green pens divided by the total number of pens: $P(\text{green}) = \frac{\text{Number of green pens}}{\text{Total number of pens}}$ $P(\text{green}) = \frac{23 - 2n}{22}$ (b) We are given that the probability of choosing a green pen is $\frac{1}{2}$. So, we can set up the equation: $\frac{23 - 2n}{22} = \frac{1}{2}$ $2(23 - 2n) = 22$ $46 - 4n = 22$ $4n = 46 - 22$ $4n = 24$ $n = \frac{24}{4}$ $n = 6$ Now that we have the value of n, we can find the number of blue pens: Number of blue pens = $n - 1$ Number of blue pens = $6 - 1$ Number of blue pens = 5	
2	(a) $\frac{2n+3+4n-1}{2n+3+4n-1+18-2n} = \frac{19}{22}$ $\frac{6n+2}{20+4n} = \frac{19}{22}$ $132n + 44 = 380 + 76n$ $56n = 336$ $n = 6 \text{ (shown)}$ (b) $\frac{15}{44} \times \frac{14}{43} = \frac{105}{946}$	
3	(a) (i) 26 (ii) Position of median = 20th and 21st (40 packets) Median = 32 (b) % packets between 24 to 32 = $\frac{14}{40} \times 100 = 35\%$	
4	(a) Paper 1 Mean = $\frac{90(5)+310(15)+370(25)+200(35)+30(45)}{1000} = 22.7 \text{ (3 s.f.)}$	

	Paper 2 Mean = $\frac{100(5)+270(15)+295(25)+295(35)+40(45)}{1000} = 24.1(3 \text{ s.f.})$ (b) Paper 1 was more difficult as the mean is lower.	
5	Possible answers: 1. No values or scale on the vertical axis (if it starts from 0 or not) - opened to misinterpretation as to the total number of passengers. OR Exaggerates the differences between the years. 2. 2017 has a different width - it is not clear whether the width or the height of the bar should be read as the total number of passenger movement. 3. 2020 bar implies the entire year while information given states up to February 2020 - distortion or misrepresentation of information.	
6	(a) For 300 children, corresponding to a full circle of (360°): $x = \frac{360}{300} \times 60$ $x = 72$ (b) Sector angle for mangoes: $360^\circ - 84^\circ - 102^\circ - 72^\circ = 102^\circ$ The number of children who like chiku to those who like mangoes: $72: 102 = 12: 17$ $\frac{102}{360} \times 100\% = 28.3\%$	
7	(i) <u>Sufyan</u>  <u>LeLe</u> D M D M <u>Legend</u> D - Dark Chocolate M - Milk Chocolate (ii)(a) $P(\text{both dark chocolate}) = \frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$	

	(b) $P(\text{LeLe picked milk chocolate}) = \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{4}{11} = \frac{5}{12}$ (c) $P(\text{at least one milk chocolate was chosen}) = 1 - \frac{7}{12} \times \frac{6}{11} = \frac{15}{22}$	
8	First, let's find the totals for the table: Total boys = $4 + 16 = 20$ Total girls = $6 + 14 = 20$ Total students who wear spectacles = $4 + 6 = 10$ Total students who do not wear spectacles = $16 + 14 = 30$ Total students = $20 + 20 = 40$ (i) (a) Find the probability that the student is a boy: $P(\text{boy}) = \frac{\text{Number of boys}}{\text{Total number of students}} = \frac{20}{40} = \frac{1}{2}$ (b) Find the probability that the student wears spectacles: $P(\text{wears spectacles}) = \frac{\text{Number of students who wear spectacles}}{\text{Total number of students}} = \frac{10}{40} = \frac{1}{4}$ (ii) Calculate the number of students to be removed so that the probability of drawing a girl who wears spectacles from the remaining students in the class is $\frac{1}{6}$. Let x be the number of students removed. We want the probability of drawing a girl who wears spectacles to be $\frac{1}{6}$. The number of girls who wear spectacles is 6. We will assume that only students who don't wear spectacles are removed. This will keep the number of girls who wear spectacles constant. So, we want: $\frac{6}{40 - x} = \frac{1}{6}$ Cross-multiply: $ \begin{aligned} 6 \times 6 &= 1 \times (40 - x) \\ 36 &= 40 - x \\ x &= 40 - 36 = 4 \end{aligned} $ So, we need to remove 4 students who do not wear spectacles to achieve the desired probability.	
9	(a) Show that $x + y = 10$: The die was thrown 20 times, so the sum of the frequencies must be 20. $ \begin{aligned} 1 + x + 4 + 3 + y + 2 &= 20 \\ 10 + x + y &= 20 \\ x + y &= 10 \end{aligned} $ (b) If the mean score is 3.7, show $2x + 5y = 44$ The mean score is calculated as $\frac{\sum(\text{score} \times \text{frequency})}{\sum \text{frequency}} = 3.7$.	

	$\frac{(1 \times 1) + (2 \times x) + (3 \times 4) + (4 \times 3) + (5 \times y) + (6 \times 2)}{20} = 3.7$ $\frac{1 + 2x + 12 + 12 + 5y + 12}{20} = 3.7$ $2x + 5y + 37 = 3.7 \times 20$ $2x + 5y + 37 = 74$ $2x + 5y = 74 - 37$ $2x + 5y = 37$ (c) Using (a) and (b), find the values of x and y: We have the system of equations: $x + y = 10$ $2x + 5y = 37$ Multiply equation (1) by -2: $-2x - 2y = -20$ Add equation (2) and (3): $3y = 17$ $y = \frac{17}{3}$ This is not an integer, which is not possible since the frequency must be an integer.	
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