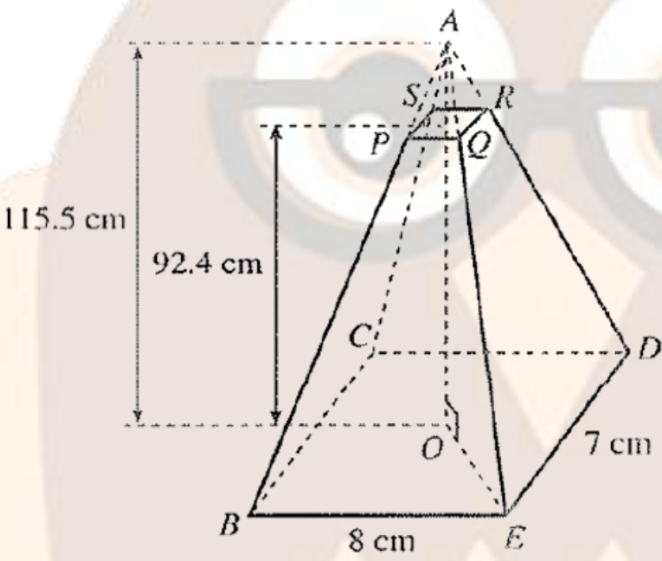
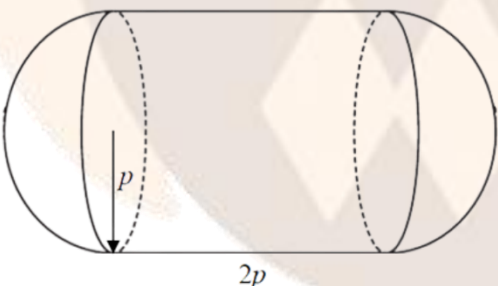
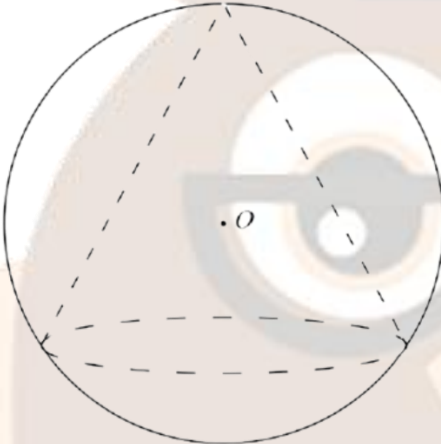
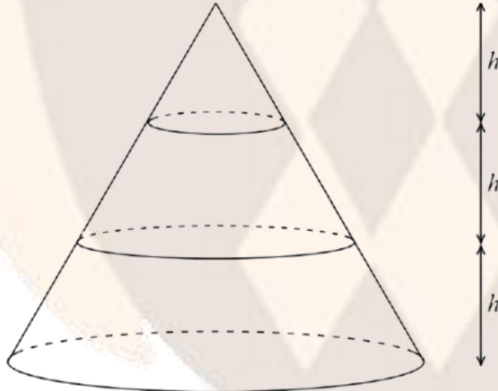
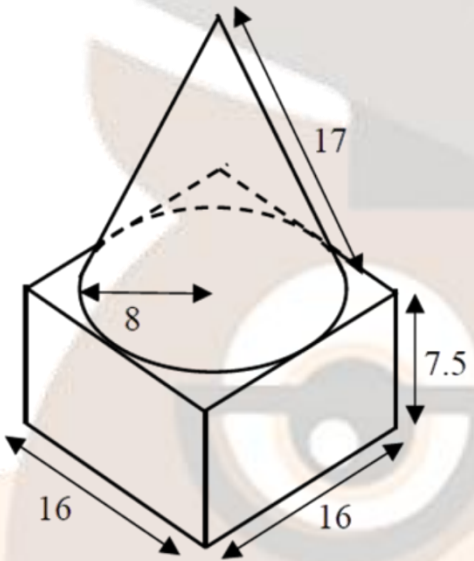


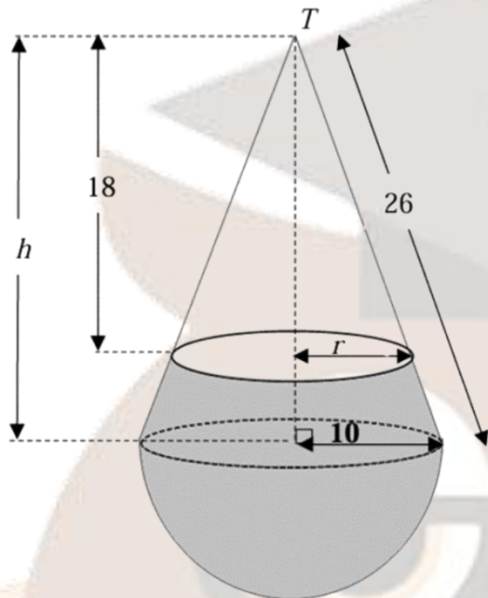
6 ESSENTIAL QUESTIONS TO MASTER

MENSURATION

1	(a) When half of a cylindrical container of radius 14 cm is filled with water, the total surface area of the container that is in contact with water is 1496 cm^2. When the container is filled to the brim, the volume of water in the container is equal to the volume of a pyramid with a rectangular base of dimension 15 cm by 8 cm. Find the height, in m, of the pyramid. (Take $\pi = \frac{22}{7}$)  (b) In the figure below, the right pyramid APQRS is cut out from the right pyramid ABEDC with base PQRS parallel to the base BEDC. Show that the length of PS is 20% the length of BC.
2	A composite solid is made from a cylinder and two hemispheres. The cylinder has radius p and length $2p$, while the hemispheres have radius p. 

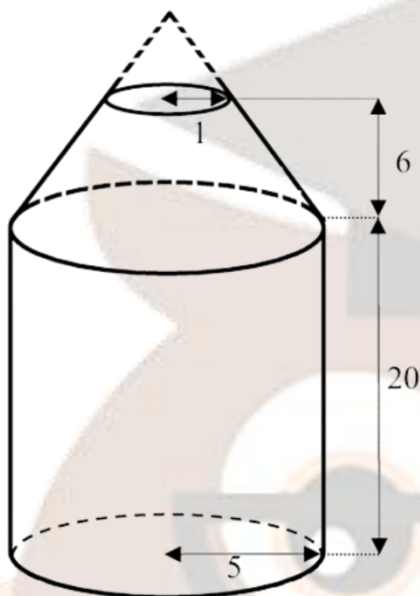
	The total surface area of the solid is six times the total surface area of a solid cone with radius p and slant height l. Find l in terms of p.	
3	The diagram shows a right circular cone cut from a solid steel sphere. Point O is the centre of the sphere with radius 12 cm. (a) Given the circumference of the base of the cone is 50 cm, show that the height of the cone is 21.0 cm, corrected to 3 significant figures. (b) Find the curved surface area of the cone.  After the cone is cut from the steel sphere, the remaining steel is melted down to form part of a solid right circular cone as shown in the following diagram. The cone comprises 3 layers of equal heights, h cm. The top and bottom layers are cast from the remaining steel. The centre section is made from acrylic.  (c) Find the volume of the acrylic used to make the centre layer of the solid cone.	

4	The diagram shows a solid formed from a cone and a cuboid. The cone has a radius of 8 cm and slant height of 17 cm. The cuboid has a square base of length 16 cm and a height of 7.5 cm.  (a) Calculate the volume of the solid figure. (b) Calculate the total surface area of the solid figure.	
5	The diagram shows a container formed from a cone and a hemisphere. The cone has base radius 10 cm, vertical height h cm and slant height 26 cm. The hemisphere has radius 10 cm. Water enters the container through a tiny hole, T at the top of the cone. The water reaches a height 18 cm from T and forms a circular top with radius r cm.	



- Find the height of the cone.
- Using similar triangles, show that $r = 7\frac{1}{2}$
- Find the surface area of the container that is in contact with the water.
- Find the volume of the water in the container.

- 6 A bottle of paint is in the shape of a frustrum of a right circular cone and a cylinder attached to it. The frustrum has a vertical height of 6 cm and a top radius of 1 cm. The attached cylinder has a radius of 5 cm and a height of 20 cm. The thickness of the bottle is negligible.



- (a) Show that the total capacity of the paint bottle is 1765.6 cm^3 , correct to 1 decimal place.
- (b) The paint bottle is filled with 1500 cm^3 , of paint. Calculate the percentage of the bottle's capacity that is not filled.
- (c) Given that the mass of the empty paint bottle is 30 g and the density of the paint is 3 g/cm^3 , find the total mass of a brand-new bottle consisting of 1500 cm^3 of paint, leaving your answer in kg.
- (d) The paint bottle is available in a similar bottle of a larger size. Given that the larger bottle has three times the capacity of the smaller bottle, find the base area of the larger bottle.

Answer Key:

1	(a) Let r be the radius of the cylindrical container, and h be its height. Given $r = 14 \text{ cm}$. When the container is half-filled with water, the total surface area in contact with water is 1496 cm^2. The surface area consists of the curved surface area, the base area, and half of the top area. So, $\pi r h + \pi r^2 + r h = 1496$.	
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Using $\pi = \frac{22}{7}$ and $r = 14$, we have:

$$\frac{22}{7} \times 14 \times h + \frac{22}{7} \times 14^2 + 14h = 1496$$

$$44h + 616 + 14h = 1496$$

$$58h = 1496 - 616$$

$$58h = 880$$

$$h = \frac{880}{58} = \frac{440}{29} \text{ cm}$$

When the container is filled to the brim, the volume of water is $V_{\text{cylinder}} = \pi r^2 h = \frac{22}{7} \times 14^2 \times \frac{440}{29} = \frac{22}{7} \times 196 \times \frac{440}{29} = 22 \times 28 \times \frac{440}{29} = \frac{270080}{29} \sim \text{cm}^3$.

The volume of the pyramid with a rectangular base of dimension 15 cm by 8 cm is equal to the volume of the water in the container.

Let H be the height of the pyramid.

$$V_{\text{pyramid}} = \frac{1}{3} \times \text{base area} \times H = \frac{1}{3} \times (15 \times 8) \times H = 40H$$

$$\text{So, } 40H = \frac{270080}{29}$$

$$H = \frac{270080}{29 \times 40} = \frac{6752}{29} \approx 232.83 \text{ cm}$$

To convert to meters, divide by 100:

$$H \approx 2.3283 \text{ m}$$

(b) Let $AO = h_1 = 92.4 \text{ cm}$ and the total height $AA' = h_2 = 115.5 \text{ cm}$.

Let $BC = x$ and $PS = y$. We need to show that $y = 0.2x$.

Since PQRS is parallel to BEDC, $\triangle APS \sim \triangle ABC$.

$$\frac{PS}{BC} = \frac{AP}{AB}$$

$$\text{We also know that } \frac{AO}{AA'} = \frac{h_1}{h_2} = \frac{92.4}{115.5} = \frac{924}{1155} = \frac{4}{5}.$$

$$\text{Since } \frac{AP}{AB} = \frac{AO}{AA'},$$

$$\frac{y}{x} = \frac{4}{5}$$

However, we are looking for $\frac{AP}{AB}$ such that PS is 20% of BC.

Since the height of pyramid APQRS is 92.4 cm and the height of pyramid ABEDC is 115.5 cm

The ratio of the height of smaller pyramid to the height of larger pyramid is

$$\frac{92.4}{115.5} = \frac{4}{5}$$

The ratio of side PS to BC is same as ratio of heights.

	$\frac{PS}{BC} = \frac{AA' - AO}{AA'} = \frac{115.5 - 92.4}{115.5} = \frac{23.1}{115.5} = \frac{1}{5} = 0.2$ So, $PS = 0.2 \times BC$	
2	1. Find the surface area of the composite solid: Cylinder: The curved surface area of a cylinder is $(2\pi r h)$, where (r) is the radius and (h) is the height. In this case, $(r = p)$ and $(h = 2p)$, so the curved surface area is $(2\pi p(2p) = 4\pi p^2)$. Hemispheres: Two hemispheres make a sphere. The surface area of a sphere is $(4\pi r^2)$. In this case, $(r = p)$, so the surface area of the two hemispheres is $(4\pi p^2)$. Total Surface Area of Composite Solid: The total surface area of the composite solid is the sum of the curved surface area of the cylinder and the surface area of the two hemispheres: $(4\pi p^2 + 4\pi p^2 = 8\pi p^2)$. 2. Find the surface area of the cone: The total surface area of a cone is $(\pi r l + \pi r^2)$, where (r) is the radius and (l) is the slant height. In this case, $(r = p)$, so the surface area of the cone is $(\pi p l + \pi p^2)$. 3. Set up the equation: The problem states that the total surface area of the solid is six times the total surface area of the cone. So, we have the equation: $8\pi p^2 = 6(\pi p l + \pi p^2)$ 4. Solve for (l): Divide both sides of the equation by (πp): $8p = 6(l + p)$ Divide both sides by 6: $\frac{4}{3}p = l + p$ Subtract (p) from both sides: $\frac{4}{3}p - p = l$ Simplify: $\frac{1}{3}p = l$ Therefore, $(l = \frac{1}{3}p)$.	
3	(a) Radius of cone $= \frac{50}{2\pi} = 7.9577cm$ (5sf) Length from O to base of cone $= \sqrt{12^2 - 7.9577^2} = 8.9819cm$ (5sf)	

	Height of cone = $12 + 8.9819 = 20.9819 = 21.0 \text{ cm}$ (3sf) (b) Slant height of cone = $\sqrt{20.9819^2 + 7.9577^2}$ or $\sqrt{21.0^2 + 7.9577^2} = 22.440 \text{ cm} = 22.457 \text{ cm}$ Curved surface area = $\pi(7.9577)(22.440) = 561 \text{ cm}^2$ (3sf) (c) Volume of cone in sphere $= \frac{1}{3}\pi(7.9577)^2(20.9819) \quad \text{or} \quad \frac{1}{3}\pi(7.9577)^2(21.0)$ $= 1391.4 \text{ cm}^3 \quad \quad \quad = 1392.6 \text{ cm}^3$ Volume of remaining steel = $\frac{4}{3}\pi(12)^3 - 1391.4$ $= 5846.8 \text{ cm}^3 \quad \text{or} \quad \frac{4}{3}\pi(12)^3 - 1392.6 = 5845.6 \text{ cm}^3$ Ratio of volumes A : A+B : A+B+C : A+C $h^3 : (2h)^3 : (3h)^3$ $1 : 8 : 27 : 20$ Volume of acrylic = $\frac{7}{20} \times 5846.8 = 2050 \text{ cm}^3$	
4	(a) height = $\sqrt{17^2 - 8^2}$ = 15 cm Volume = $\frac{1}{3}\pi(8)^2(15) + (16 \times 16 \times 7.5)$ = 2925.3 = 2930 cm^3 (3sf) (b) Total SA = $\pi(8)(17) + [(16 \times 16) - \pi(8)^2] + [(16 \times 16) + (4 \times 16 \times 7.5)]$ = 1218.1 = $1220 \sim \text{cm}^2$ (3sf)	
5	(a) 24 cm	

	(b) $\frac{r}{10} = \frac{18}{24}$ $r = 7\frac{1}{2}$ (c) Surface area = SA of hemisphere + SA of cone $= 2\pi(10)^2 + 26\pi(10) - \pi(7.5)\sqrt{18^2 + 7.5^2}$ $= 986 \text{ cm}^2$ (d) Volume = Vol of hemisphere + Volume of frustrum $= \frac{2}{3}\pi(10)^3 + \frac{37}{64} \times \frac{1}{3}\pi(10)^2 \times 24$ $= 3550 \text{ cm}^3$	
6	(a) Let x be the height of the cone that was removed. By similar triangles, $\frac{x}{x+6} = \frac{1}{5}$ $5x = x+6$ $4x = 6$ $x = 1.5$ Volume of paint bottle: $= \pi(5^2)(20) + \left(\frac{1}{3}\pi \times 5^2 \times 7.5\right) - \left(\frac{1}{3}\pi \times 1^2 \times 1.5\right)$ $= 1765.5755$ $= 1765.6 \text{ cm}^3 \text{ (shown)}$ (b) $\frac{1765.6 - 1500}{1765.6} \times 100$ $= 15.043$ $= 15\%$ (c) $= 30 + 1500(3)$ $= 4530 \text{ g}$ $= 4.53 \text{ kg}$ (d) $\frac{V_L}{V_S} = \frac{3}{1}$ $\frac{l_L}{l_S} = \sqrt[3]{\frac{3}{1}}$	

$\frac{A_L}{A_S} = \left(\frac{3}{1}\right)^{\frac{2}{3}}$ Base area of larger bottle $= \left(\frac{3}{1}\right)^{\frac{2}{3}} \times \pi(5)^2$ $= 163.3694$ $= 163 \sim \text{cm}^2 (3\text{s.f})$	
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