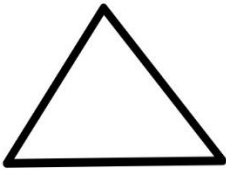
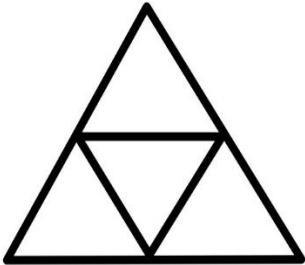
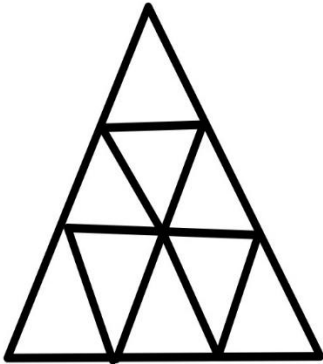


6 ESSENTIAL QUESTIONS TO MASTER

NUMBER PATTERNS

1	The first four terms of a sequence are 1, 2, 5, and 10. (a) Write down the next two terms. (b) Find an expression, in terms of n, for the nth term of the sequence. (c) Explain, with working, why 102 is not a term in the sequence.	[1] [2] [2]
2	A sequence of patterns formed by dots is as shown. <u>Pattern 1</u>  Number of dots = 1 <u>Pattern 2</u>  Number of dots = 5 <u>Pattern 3</u>  Number of dots = 9 <u>Pattern 4</u>  (a) Draw the 4th pattern of the sequence in the above box. (b) State the total number of dots required, in terms of n, to form the nth pattern? (c) Pattern P requires 357 dots. What is the value of 'P'?	[1] [2] [2]
3	Given the following sequence, $\frac{1}{2} + \frac{1}{2} = 1$ $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$ $\frac{1}{12} + \frac{1}{4} = \frac{1}{3}$ $\frac{1}{20} + \frac{1}{5} = \frac{1}{4}$... $\frac{1}{p} + \frac{1}{11} = \frac{1}{10}$ Find (a) the 6th line of sequence (b) the value of (p) (c) the value of $(\frac{1}{100} - \frac{1}{101})$, showing your workings clearly.	[2] [1] [3]

4	The following diagram shows the first three figures of a sequence of triangles.  Figure 1 1 Triangle  Figure 2 4 Triangles  Figure 3 9 Triangles <table border="1" style="margin-top: 20px; width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Figure</th><th>No of Triangles</th><th>Total</th></tr> </thead> <tbody> <tr> <td>1</td><td>1</td><td></td></tr> <tr> <td>2</td><td>1+3</td><td></td></tr> <tr> <td>3</td><td>1+3+5</td><td></td></tr> <tr> <td>4</td><td></td><td></td></tr> <tr> <td>5</td><td></td><td>x</td></tr> <tr> <td>...</td><td>...</td><td>...</td></tr> <tr> <td>n</td><td>1+3+5+...+y</td><td>z</td></tr> </tbody> </table> (a) Complete the table above to find the value of x. [4] (b) Write an expression for y and z, in terms of n, giving your answers in their simplest form. [2] (c) Is it possible for a figure to consist of 4119 triangles? [2]	Figure	No of Triangles	Total	1	1		2	1+3		3	1+3+5		4			5		x	...	...	...	n	1+3+5+...+y	z	
Figure	No of Triangles	Total																								
1	1																									
2	1+3																									
3	1+3+5																									
4																										
5		x																								
...	...	...																								
n	1+3+5+...+y	z																								
5	The birth length of a newborn baby boy is 50 cm. During the first year, his length increases by 2.5 cm every month. (a) Write down his length when he is (i) 1 month old, [1] (ii) 2 months old. [1] (b) Find an expression for his length when he is n months old. [1] (c) If the boy's length is 72.5 cm when he is m months old, find the value of m. [2] (d) Explain why the expression in (b) is not used to find the length of the boy when he is 5 years old. [2]																									
6	The First three terms in a sequence of numbers, $U_1, U_2, U_3, \dots$ are give below: $U_1 = 2 + 4 = 6$ $U_2 = 6 + 6 = 12$ $U_3 = 12 + 8 = 20$ (a) Find U_4 [1] (b) Find an expression, in terms of n, for U_n. [2]																									

	Evaluate U_{25}	[1]
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Answer Key:

1	(a) Let's look at the differences between consecutive terms: $2 - 1 = 1$ $5 - 2 = 3$ $10 - 5 = 5$ The differences are increasing by 2 each time. This suggests that the next differences will be 7 and 9. So, the next two terms are: $10 + 7 = 17$ $17 + 9 = 26$ Therefore, the next two terms are 17 and 26. (b) Notice that the terms can be written as: * $1 = 0^2 + 1$ * $2 = 1^2 + 1$ * $5 = 2^2 + 1$ * $10 = 3^2 + 1$ So, the n th term can be expressed as $(n - 1)^2 + 1$. Let's denote the n th term as a_n. Then, $a_n = (n - 1)^2 + 1$ (c) If 102 is a term in the sequence, then there must be an integer n such that $a_n = 102$. So, we need to solve the equation: $(n - 1)^2 + 1 = 102$ $(n - 1)^2 = 101$ $n - 1 = \pm\sqrt{101}$ $n = 1 \pm \sqrt{101}$ Since $\sqrt{101}$ is not an integer, n is not an integer. Therefore, 102 is not a term in the sequence.	
2	(a)	



(b)

$$T_1 = 4(1) - 3$$

$$T_2 = 4(2) - 3$$

$$T_3 = 4(3) - 3$$

$$T_4 = 4(4) - 3$$

$$T_n = 4n - 3$$

(c)

$$Tp = 4p - 3 = 357$$

$$4p = 360$$

$$p = 84$$

3

(a) For the 6th line, $n = 6$. So,

$$\frac{1}{6(6+1)} + \frac{1}{6+1} = \frac{1}{6}$$

$$\frac{1}{6(7)} + \frac{1}{7} = \frac{1}{6}$$

$$\frac{1}{42} + \frac{1}{7} = \frac{1}{6}$$

$$\frac{1}{42} + \frac{1}{42} = \frac{2}{42} = \frac{1}{21}$$

(b) We are given $\frac{1}{p} + \frac{1}{11} = \frac{1}{10}$.

$$\frac{1}{p} = \frac{1}{10} - \frac{1}{11} = \frac{11-10}{110} = \frac{1}{110}$$

(c)

$$\frac{1}{100} - \frac{1}{101} = \frac{101 - 100}{100 \times 101} = \frac{1}{10100}$$

4

Figure	No of Triangles	Total
1	1	1
2	1+3	4
3	1+3+5	9
4	1+3+5+7	16
5	1+3+5+7+9	25
...	...	...

	n $1+3+5+\dots+y$ z (a) $x=25$ (b) $y=2n-1$ $z=n^2$ (c) No, $\sqrt{4119} = 64.1794$, which is not a whole number. OR 4119 does not give a whole number as answer.	
5	(a)(i) $50 + 2.5(1) = 50 + 2.5 = 52.5$ cm (ii) $50 + 2.5(2) = 50 + 5 = 55$ cm (b) $L = 50 + 2.5n$ (c) $72.5 = 50 + 2.5m$ $22.5 = 2.5m$ $m = \frac{22.5}{2.5} = 9$ (d) The expression in (b) is not used to find the length of the boy when he is 5 years old because the growth rate of 2.5 cm per month is only applicable during the first year of his life. After the first year, the growth rate will likely change and is not constant. The baby's growth rate slows down after the first year.	
6	The First three terms in a sequence of numbers, $U_1, U_2, U_3, \dots$ are give below: $U_1 = 2 + 4 = 6$ $U_2 = 6 + 6 = 12$ $U_3 = 12 + 8 = 20$ (a) Find U_4 - Observe the pattern: $U_1 = 2 + 4 = 6$ $U_2 = 6 + 6 = 12$ $U_3 = 12 + 8 = 20$ - The first number being added is the previous term. The second number being added increases by 2 each time (4, 6, 8, ...). So, the next term will be 10. - Therefore $U_4 = 20 + 10 = 30$ (b) Find an expression, in terms of n, for U_n. - Let's Analyze the sequence of terms: 6, 12, 20, ... - Look at the differences between consecutive terms: $12 - 6 = 6$, $20 - 12 = 8$. The differences are increasing by 2. This suggests a quadratic relationship. - Let's assume $U_n = an^2 + bn + c$ - Using the first three terms: $U_1 = a(1)^2 + b(1) + c = a + b + c = 6$ $U_2 = a(2)^2 + b(2) + c = 4a + 2b + c = 12$	

	• $U_3 = a(3)^2 + b(3) + c = 9a + 3b + c = 20$ • Solve this system of equations. Subtract the first equation from the second and the second from the third: • $3a + b = 6$ • $5a + b = 8$ • Subtract the first equation from the second: • $2a = 2$ • $a = 1$ • Substitute $a = 1$ into $3a + b = 6$: • $3(1) + b = 6$ • $b = 3$ • Substitute $a = 1$ and $b = 3$ into $a + b + c = 6$: • $1 + 3 + c = 6$ • $c = 2$ • Therefore $U_n = n^2 + 3n + 2$ • Factor this expression: $U_n = n^2 + 3n + 2$ or $(n + 1)(n + 2)$ Evaluate U_{25} • Substitute $n = 25$ into the expression for U_n: • Or using the factored form: $U_{25} = (25 + 1)(25 + 2) = (26)(27) = 702$	
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