

7 ESSENTIAL QUESTIONS TO MASTER

HCF & LCM

1	(a) Written as a product of its prime factors, $500=2^x \times 3^y \times 5^z$ Find the values of x, y and z. (b) Is 500 a perfect cube? You may use your answer in part (a) to explain why.	[2] [2]
2	(a) Find (k) such that $(84k)$ is both a perfect square and a perfect cube. (b) Express 30 as a product of its prime factors. Give your answer in index notation. (c) Find the highest common factor (HCF) of 30 and 84. (d) Two alarm clocks are set to ring at intervals of 30 minutes and 42 minutes, respectively. If the alarm clocks ring together at 0900, at what time will they next ring together again?	[1] [1] [1] [2]
3	(a) Express 360 as a product of its prime factors. (b) Find the smallest possible integer value of n such that $360n$ is a multiple of 48.	[1] [1]
4	(a) Find the highest common factor of 1540 and 2800. (b) Find the lowest common multiple of $2^4 \times 3^3 \times 5^2$ and $2^5 \times 3 \times 7^2$. Leave your answer in index notation form. (c) Using prime factorisation, find $\sqrt{2025}$. Show your working clearly. (d) Find the smallest integer value of k where $180k$ is a perfect square.	[1] [1] [2] [2]
5	(a) Express 576 as a product of its prime factors. (b) Using your answer in (a) explain why 576 is a perfect square. (c) Find the largest integer that can divide both 576 and 48. (d) Given that $576m$ is a multiple of 60. Find the smallest possible integer value of m.	[1] [1] [1] [1]
6	The number 144, written as the product of its prime factors, is $2^4 \times 3^2$. (a) Express 180 as a product of its prime factors, giving your answer in index notation. (b) Find the smallest non-zero whole number that is divisible by both 144 and 180. (c) a and b are both prime numbers. Find the values of a and b so that $140 \times 180 \times \frac{a}{b}$ is a perfect cube.	[2] [1] [2]
7	1530 bottles of water, 680 energy bars, and 1020 sanitizing wipes were collected in a charity drive for disaster relief. The items were packed into relief kits for families. Each kit has the same number of bottles of water, energy bars, and sanitizing wipes. Find the greatest number of relief kits that can be packed.	[3]

Answer Key:

1	(a) Divide by the smallest prime (2): $(540 \div 2 = 270)$ Continue dividing by 2: $(270 \div 2 = 135)$ (Here 135 is odd, so we stop dividing by 2.) Next smallest prime (3): $(135 \div 3 = 45)$ $(45 \div 3 = 15)$ $(15 \div 3 = 5)$ (At this point, we have reached the prime number 5.) Final prime: $(5 \div 5 = 1)$ $(x = 2)$ (for (2^x)) $(y = 3)$ (for (3^y)) $(z = 1)$ (for (5^z)) (b) Since not all the exponents are multiples of 3, 540 is not a perfect cube.	
2	(a) $84 = 2^2 \times 3^1 \times 7^1$ 2^4 needs 2^2 more to become 2^6. 3^1 needs 3^5 more to become 3^6. 7^1 needs 7^5 more to become 7^6. Hence $k = 2^4 \times 3^5 \times 7^5$ (b) Prime Factorization of 30 [$30 = 2 \times 3 \times 5$] (c) Finding HCF: Common primes: (2) and (3) Calculating HCF: [2×3] Final Answer for (c): HCF = 6 (d) Finding LCM of 30 and 42: Prime factorizations: $30 = 2 \times 3 \times 5$ $42 = 2 \times 3 \times 7$ Calculating LCM: $2 \times 3 \times 5 \times 7 = 210$ minutes (3h 30min) Next Time Together: Initial time: 0900 Add 3 hours and 30 minutes: $0900 + 0330 = 1230$ Final Answer for (d): They will next ring together at 1230.	



3	(a) $360 = 2^3 \times 3^2 \times 5$ (b) $360 = 2^3 \times 3^2 \times 5$ 360 has 2^3 but 48 requires 2^4 360 has 3^2 but 48 requires 3 360 has 5^1 which is not needed. Hence, we need an additional 2^1 to make the power of 2 sufficient for 48. The smallest value of n is $2^1=2$, since that will ensure that $360n$ contains the prime factorisation of 48. Therefore, the smallest possible integer value of n is 2.	
4	(a) $1540 = 2^2 \times 5 \times 7$ $2800 = 2^4 \times 5^2 \times 7$ $\text{HCF}(1540, 2800) = 2^2 \times 5^1 \times 7^1 = 140$ (b) $\text{LCM} = 2^5 \times 3^3 \times 5^2 \times 7^2$ (c) $2025 = 3^4 \times 5^2$ $\sqrt{2025} = \sqrt{3^4 \times 5^2} = 3^2 \times 5 = 45$ (d) To make $180k$ a perfect square, all prime powers must be even. Increase the power of 5 to make it even. Therefore $k = 5$	
5	(a) $576 = 2^6 \times 3^2$ (b) Since all the exponents in the prime factorization of 576 are even, 576 is a perfect square. $\sqrt{576} = 2^3 \times 3 = 24$ (c) $576 = 2^6 \times 3^2$ $48 = 2^4 \times 3$ $\text{HCF}(576, 48) = 2^4 \times 3$ (Largest Integer) (d) Given that $576m$ is a multiple of 60, Find the smallest possible integer value of m. $60 = 2^2 \times 3 \times 5$ 576 already contains 2^6 and 3^2, but lacks 5. Therefore, m must be at least 5 to include this factor. Hence, the smallest possible integer value of m is 5.	
6	(a) $180 = 2^2 \times 3^2 \times 5$ (b) $\text{LCM}(144, 180)$ $2^4 \times 3^2 \times 5^1 = 720$	



	(c) Let's start with the prime factorizations of 140 and 180: Prime factorization of 140: $2^2 \times 5^1 \times 7^1$ Prime factorization of 180: $2^2 \times 3^3 \times 5^1$ Combining the factorizations: $140 \times 180 = 2^4 \times 3^2 \times 5^2 \times 7^1$ For the expression $140 \times 180 \times \frac{a}{b}$ to be a perfect cube, all exponents in its prime factorization must be multiples of 3. Current exponents: 2^4 needs 2^2 more to become 2^6. 3^2 needs 3^1 more to become 3^3. 5^2 needs 5^1 more to become 5^3. 7^1 needs 7^2 more to become 7^3. We need to adjust these exponents using the prime numbers a and b. Let's assign a and b such that: a=7 (to contribute 7^1) b=5 (to contribute 5^{-1}) This gives us: $140 \times 180 \times \frac{7}{5} = 2^4 \times 3^2 \times 5^1 \times 7^2$ Now, adjust: 2^4 is already 2^6 after considering a and b. 3^2 is already 3^3. $5^1 \times 5^{-1} = 5^0$ is neutral. $7^2 \times 7^1 = 7^3$ Therefore, a=7 and b=5 satisfies the condition for making the product a perfect cube.	
7	The prime factorizations are: - 1155: $(3 \times 5 \times 7 \times 11)$ - 462: $(2 \times 3 \times 7 \times 11)$ - 924: $(2^2 \times 3 \times 7 \times 11)$ The common prime factors are (3), (7), and (11). Multiply the smallest power of each common factor: $\text{GCD} = 3^1 \times 7^1 \times 11^1 = 231$ Therefore, the greatest number of gift bags that can be packed, with each bag containing the same number of each item, is (231).	